Construction of mathematical knowledge using graphic calculators (TI-84 plus & CAS) in the mathematics classroom

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ABSTRACT

Mathematics education researchers are asking themselves about why technology has impacted heavily on the social environment and not in the mathematics classroom. The use of technology in the mathematics classroom has not had the expected impact, as it has been its use in everyday life (f.e. cell phone). What about teachers' opinion? Mathematics teachers can be divided into three categories: Those with a boundless overflow (enthusiasm) that want to use the technology without worrying much about the construction of mathematical concepts, those who reject outright the use of technology because they think that their use inhibits the development of mathematical skills, and others that reflect on the balance that must exist between paper-pencil activities and use of technology. The mathematics teacher not having clear examples that support this last option about the balance of paper-pencil activities and technology, opt for one of the extreme positions outlined above. In this paper, we show the results of research on a methodology based on collaborative learning (ACODESA) in the training of mathematics teachers in secondary schools and implementation of activities in an environment of paper-pencil and CAS in the mathematics classroom. We note also that with the development of technology on the use of electronic tablets and interactive whiteboards, these activities will take on greater momentum in the near future.

RESUMEN

Los investigadores en educación matemática se preguntan por qué la tecnología ha impactado fuertemente en el medio social y no en el aula de matemáticas. El uso de la tecnología en el aula de matemáticas no ha tenido el impacto esperado como lo ha sido su uso en la vida corriente. ¿Cuál es la posición de los profesores? Los profesores de matemáticas se pueden dividir en tres tipos de categorías, aquellos que con un desbordamiento desmedido quieren utilizar la tecnología sin preocuparse mucho sobre la construcción de conceptos, aquellos que rechazan

completamente el uso de tecnología porque piensan que su uso inhibe el desarrollo de habilidades matemáticas, y otros que reflexionan sobre el equilibrio que debe de haber entre actividades en papel-lápiz y uso de tecnología. El profesor de matemáticas al no contar con ejemplos claros que soporten esta última opción sobre el equilibrio en las actividades de papel-lápiz y tecnología, opta por alguno de los extremos antes señalados. En este documento, queremos mostrar los resultados de investigación sobre una metodología basada en el aprendizaje en colaboración (ACODESA) en la formación de profesores de matemáticas en la escuela secundaria e implementación de actividades en ambientes de papel-lápiz y calculadora (CAS) en el aula de matemáticas. Queremos señalar también, que con el desarrollo de la tecnología sobre el uso de tabletas electrónicas y pizarrones interactivos, este tipo de actividades cobrarán un impulso mayor en un futuro inmediato.

Introduction

The use of technology in the mathematics classroom, for one reason or another, has failed to permeate the mathematics classroom. This, bearing in mind that the educational authorities are aware of the importance of using technology in the classroom. For example, in Québec, the Ministère de l'Education, du Loisir et du Sport (MELS, 1996) proclaims the importance of its use in the mathematics classroom, however, as it presented to the teacher of mathematics is naive, and even dangerous.

> Because the technology influences on mathematics and its use, is necessary for the student to control modern electronic tools like the scientific calculator...
> The technology does not guarantee student's success in mathematics; calculators and computers, as word processing for a writer, are nothing more than tools…
> **However, it enables students to acquire and understand new concepts quickly.** [Highlighted by us]. (p. 6)

We can put the educational authorities at the same level that some maths teachers who believe that everything is possible with the technology and concepts are acquired quickly and easily...

Researchers in didactics of mathematics, who believe that technology is important in the development of mathematics, have conducted experiments that put us on guard against the ingenuity of the authorities and some teachers of mathematics with respect to the direct use of technology without didactic considerations. For example, Guin et Trouche (1999, p. 195-196), stated as follows:

> No more than 15% of teachers include graphing calculators in their teaching, despite the fact that all students have a graphing calculator in science classes [pre-university level]. Teachers have a tendency to oppose even the integration of new technologies at the elementary level.

Also, they stated (Idem, p. 191) that, two groups of 50 students (pre-university students) were asked about the following limit: $\lim_{x \to \infty} ln\ x + 10\ sin\ x = \infty$? One group

using calculator: 75% of success; and the other one without calculator: 95% of success.

Let us show another research conducted by Tall (2000, p. 213), with two groups, one using DERIVE and the other without technology. The students of the two groups (pre-university level) were asked to provide a conceptual explanation of: $\lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$. The results were that with DERIVE: 0% of success; and, the No-DERIVE group: 100% of success.

These authors, Guin, Trouche, Tall, among others, are trying to show that the processes of instrumentation and instrumentalization (ROBARDEL, 1995) are not considered as complex processes that must be treated carefully in the mathematics classroom.

From our point of view, the discussion should not be whether or not we agree to use technology. Rather, if we foster the use of technology, we as Guin, Trouche & Tall (among others), must conduct research that allows us to provide more clearly the advantages and disadvantages of using technology in the mathematics classroom to improve teaching strategies.

This introduction to the problems of using technology in the mathematics classroom gives us a better idea about how to move forward on teaching strategies to improve mathematics learning in technological environments. A reflection is needed about:

1) On the processes of instrumentation and instrumentalization that should be promoted in the mathematics classroom when dealing with technological artifacts,
2) On the tasks and actions that should be promoted in the mathematics classroom in a technological context. Not all problems that were used in the past serves in a technological context (at least it needs to be adapted to the environment in which they are to be worked).
3) The attention to the balance that should exist between activities in a pencil and paper environments and actions from a technological standpoint,

If we look, in particular, on a theory of representation, and in general, in a transition from a constructivist theory to a social-interactionist theory of learning mathematics, we should consider various aspects such as follows.

From a constructivist perspective (individual construction of knowledge) we have to consider the institutional representations (those that appear in textbooks, on computer and calculators screens', etc):

A) A mathematical representation is partial with respect to what they represent (Duval, 1993, 1995).
B) A mathematical concept is constructed as an articulation among the representations of the concept under construction (Duval, 1993, 1995). Therefore, in the construction of a concept, each representation of a concept needs to be considered at the same level of education without giving a

priority to any of them. And processes of conversion among representations play a special role in the construction of the concept.

From an interactional point of view, functional representations embedded in a design approach and analysis about the action needed to develop learning in a social construction of knowledge is important. That is:

A) Design of the task and an analysis of the action (Leontiev, 1984, Hitt & Kieran, 2009) associated with it in a social environment of knowledge construction.

B) The non-institutional representations (diSessa, 1991, Hitt & Morasse, 2009) linked to the action of the individual against an argument with another team member in the mathematics classroom and analysis of its evolution towards institutional representations within an atmosphere of social construction of knowledge.

C) Technical processes and conceptualization of mathematics in technological environments and social interaction.

While theoretical aspects about representations rose more than a century ago, a theory of representations from a constructivist point of view and linked to mathematics education was developed from 1980 to 2000. The issues dealt with were heavily focused on building institutional representations of knowledge from an individualist point of view. Also theories about the construction of knowledge from a social point of view emerged from early last century and gradually theoretical aspects of representations have been considered under these approaches on the social construction of knowledge. For example the work of Bourdieu (1980) with his notion of ***habitus*** reproduces the theory developed by Vygotsky, Luria & Lontiev among others with an approach that considers the internal and external representations as important to build in social-interactionist environments.

In the context of construction of mathematical concepts related to the representations in a social interaction, we can mention diDessa et al. (1991), Gonzalez et al., (2008), Hitt & Morasse (2009) who analyzed a construction of mathematical concepts from a social-interactionist point of view. It is in the approach of these authors, among others, where we can see more clearly the importance of designing mathematical tasks and on the analysis of actions students did when solving mathematical problem situations. In Hitt & Cortes (2009) and Hitt & Kieran (2009) they consider activities in a social-interactionist setting that involve technical processes and conceptualization both in an environment of paper-pencil and calculator.

Research questions in relation to teaching of mathematics in a socio interactionist approach of learning

In a first approach to the problem, from a methodological point of view, we were concerned to the following (Hitt, 2007, Hitt & Cortes, 2009):

- What methodology is appropriate in an environment of social interaction in the learning of mathematics?

- How to integrate collaborative learning, scientific debate and self-reflection (ACODESA methodology) in the mathematics classroom?
- What is the role of the calculator when solving problem situations in a social environment of construction of knowledge?

Articulation among representations

In what follows, we consider a project that gave birth to the ACODESA methodology in a CAS environment (Hitt, 2007). In an approach on the use of different representations we proposed, to a population of secondary teachers in service who were studying a master degree of didactics of mathematics (methodology used during 2 semester courses), an activity which at first we thought it was simple and could be an exercise that would allow the introduction of processes for converting from one representation to another.

Students in this activity were already familiar to work in teams according and to discuss with the whole class, according to the ACODESA methodology (see Hitt 2007), which in brief considers five stages:

- Individual work: production of functional representations to understand the task;
- Teamwork on the same task. Process of discussion and validation. Refinement of functional representations;
- Discussion (could became a scientific debate). Process of discussion and validation (refinement of representations);
- Back on the task individually (individual work: reconstruction and self - reflection);
- Institutionalization. Processes of institutionalization and use of institutional representations.

The activity was:

Calculate the derivative of the function: $f(x)=\begin{cases} x^2 \sin\left(\frac{1}{x}\right) & si \quad x \neq 0 \\ 0 & si \quad x = 0 \end{cases}$.

The team work was carried out as usual, the teacher could identify, by looking at the production of students, who either did not know what to do to get a derivative of a piecewise function; or, as it happened with a large majority of students, their products were related to a conception that consist in: "Given a piecewise function, the derivative of the function is calculated directly by differentiating each of the expressions that define the function." That is, in our case, the calculation of the derivative of the piecewise function was the result: $f'(x)=\begin{cases} 2x \sin\left(\frac{1}{x}\right) - \cos\left(\frac{1}{x}\right) & si \quad x \neq 0 \\ 0 & si \quad x = 0 \end{cases}$

The teacher found that students did not perform any conversion among representations. Students indicate that there was no discussion and that the activity was a simple exercise about differentiation. The teacher asked one of the teams to show their results to the rest of the class.

A student (a teacher in service, named Wendy), mentions only that the derivative of the piecewise function provides the result above indicated.

The teacher requested further arguments, and another student (a teacher in service, named Lidia), who we can say she is a formalist, mentioned verbally that the derivative of the piecewise function was related to the calculation of the derivative of the 1st term, and then, the calculation of the lateral limits as x tends to 0. Finally, she obtained the same result as Wendy, but she provided a different argument.

The teacher asked Lidia to write down on the blackboard what she was saying. Lidia realizes that something was wrong because it was not as immediate that $\lim_{x\to 0}\left(2x\sin\left(\frac{1}{x}\right)-\cos\left(\frac{1}{x}\right)\right)=0$. In fact, Lidia mentioned that $\lim_{x\to 0}\left(2x\sin\left(\frac{1}{x}\right)\right)=0$, but $\lim_{x\to 0}\left(-\cos\left(\frac{1}{x}\right)\right)\neq 0$ and as a consequence, she cannot confirm that the limit was 0. Indeed, she has doubts about what could be the derivative of the function at zero.

Once Lidia came back to her seat, another student (teacher in service, named Victor), affirmed that, given Lidia's analysis, the derivative at zero DOES NOT EXIST.

Scientific debate emerged in the mathematics classroom

Wendy argues that the derivative at x = 0, is 0. This time, the argument provided was a graphical representation obtained with her calculator, the graphic representation of the function "does not oscillate at x = 0". The teacher asked her to show the result to her teammates, and she showed what is in Figure 1.

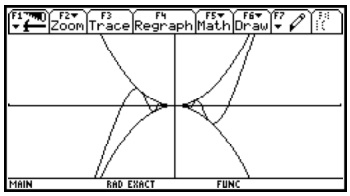

Figure 1. Screen showed by Wendy

Alicia, on the contrary, believes that the derivative does not exist "No, the function ***has*** so many oscillations", as Victor claimed.

Victor notes that, in general, is not convenient to have full confidence to the calculator; because the calculator can provide a nice graphic, but is not really so. It adds that: "... there really are many oscillations and is not that (the graph) that has a nice behavior."

The majority believed that Lidia is right (that a derivative must exist at x = 0, but different to zero) and Victor and Wendy were wrong. Wendy under this flow of opinions mentioned then: "I think I'm the only one to say that the derivative at 0 is 0, they may be right." The course ends, students and teachers continue the discussion for 20 minutes without really providing another argument.

The teacher mentioned that given the arguments put forward and the lack of consensus, according to ACODESA methodology, suggests the need for reflection at home and that the next course will recommence the activity.

In the following course, Lidia begun the discussion: "The derivative of this function [f (x)] (see Figure 2), is equal to 0 for x = 0; however, there is no way that this function is

continuous!" Lidia used the calculator to show that it is not possible to decide on the derivative at x = 0: "The graph has many oscillations, there, close to 0, therefore, I have the impression that you can not glue the zero! The graph is a source of information, but the graph is not reliable when we do zooms, the graph shows oscillations and a vertical line."

This means that Lidia did zooms around (0, 0). Adding emphatically that the function is discontinuous at zero *a fortiori*.

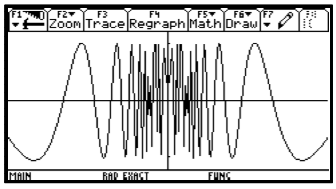

Figure 2. Lidia's graphic representation

During the discussion, Wendy uses her calculator all the time, and then the teacher asked about her opinion. Wendy stated that the result she had obtained that the derivative at 0 is 0, is true. Then she went to the blackboard to show a new algebraic argument:

$$\lim_{h\to 0}\frac{f(0+h)-f(0)}{h}=\lim_{h\to 0}\frac{h^2\sin\left(\frac{1}{h}\right)-0}{h}=\lim_{h\to 0}h\sin\left(\frac{1}{h}\right)=0$$

Wendy's argument is correct, but all doubts expressed before emerged and the majority disagrees. Some even try to find an error in her procedure. For example, Irene says, "I'm not sure, but for h = 0, we have something like *sinus* of the infinite."

We can see that Irene's conception is like the conception of the majority of high school and some college students have that when we are saying that "*h tends to zero*" they think that "finally", "*it means that h = 0*". This conception can also be detected in some teachers (as Irene).

Victor intervened likely to insist that an error must exist in the algebraic treatment, he says: "If we accept the result [Wendy's] as true, then we must accept that $\lim_{x\to 0}\left(2x\sin\left(\frac{1}{x}\right)-\cos\left(\frac{1}{x}\right)\right)=0$ What is impossible! There is a clear contradiction. "

Wendy, always using her calculator, responds with another argument, but now a visual one; she says: "but it is true that the derivative of this function is 0, ISSS 0, graphically is the horizontal line." Then she says that in any case, she wanted them to show her where the error was in her algebraic approach. She did not get a response to her request...

During the discussion, the majority of the students used the calculator. Victor came to the front of the classroom and connected his calculator to show different windows using the view screen of the calculator (see Figure 3)...

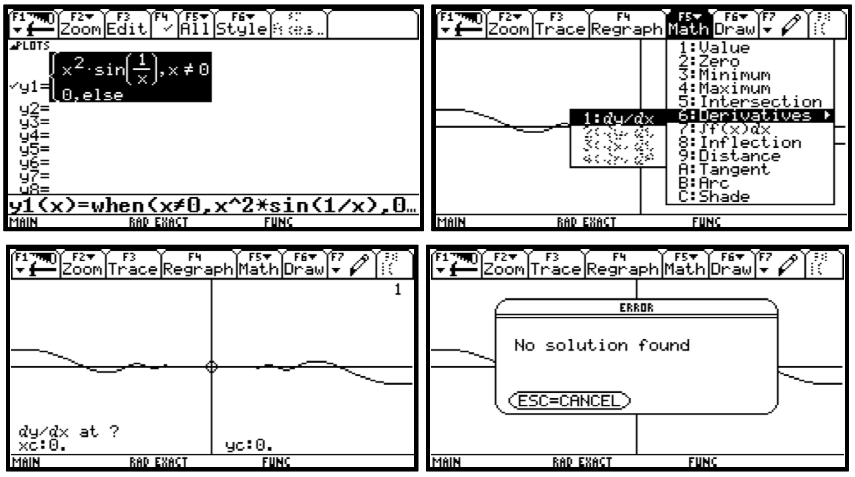

Figure 3. Victor's CAS production

This leads to a complete destabilization of the students...

Wendy reacted to defend herself... saying that should not have full confidence to the calculator ... (she used exactly the same arguments given by Victor). Indeed, she showed a summary of several cases when the calculator provided inaccurate and/or false results during the course.

We have reached a very important point in this scientific debate. Victor mentions something that is a key to solve the conflict, the articulation among representations, he mentions that surely must be an error in the algebraic treatment, but at the same time; he says, he could not really reconcile the visual results showed by the calculator with the algebraic treatment.

Again the lesson has finished, but this time the students are discouraged, a feeling of powerlessness prevails in the classroom, and students seek the answer to the teacher. The teacher reminds them that the contract was that immersed in the ACODESA methodology; it is their duty to confirm or not the results. The teacher asked them again to reflect at home...

The discussion begun again in the following lesson, the division among students was patent. However, some minutes after the discussion was initiated, Victor said: "The derivative of a differentiable function ***is not necessarily continuous***," Then, at that moment, everyone understood that the error was in the belief that "The derivative of a differentiable function is necessarily continuous," believe that implicitly Lidia transmitted to all of them. Victor mentions that the whole problem was in that interpretation, and what Wendy did, was right!

First, we can see that in a problem solving process, Duval's theoretical framework does not allow an analysis of the situation. In this case, it is not only conversion among representations; it is about algebraic arguments assembled with visual arguments in relation to a correct resolution free of contradictions. The situation requires to be aware of the contradiction and then to perform a mathematical process and coordination between representations to solve the contradiction. This could be possible because the collaborative environment of learning. Students made a considerable effort to discover that (two lessons an a half, and reflections at home). In fact, students were facing a cognitive obstacle given by a conception: "If a function is differentiable then its derivative is continuous." The contradiction was solved and in fact they had a counter example of that proposition, surpassing the cognitive obstacle.

Research questions in relation to a design of activities for 10th grade secondary students

Again immersed in a CAS environment, our research questions about a specific design of activities for secondary students, had to do with (see Hitt & Kieran, 2009):

What is the role of the technique in the conceptualization of mathematics?

Is there an interaction between technique and conceptualization?

Can we speak of a conceptual understanding of a technique?

Activities for secondary students (10th grade)

In a new project led by Kieran at the Université du Québec à Montréal (UQAM), the team designed activities for 10th grade secondary students, in CAS environments. Eight activities were designed, and researchers asked several high school teachers' opinions on the use of these activities. Some were modified according to their suggestions and finally reached the final version (see http://www.math.uqam.ca/APTE/Taches.html).

Methodology

In relation with this project, different experimentations were designed, included experimental groups of different countries:

- 3 classes (two in English-speaking school, and other French speaking) in Montreal, Canada, during the period from Sept-Feb (2004-2005);
- 1 class in Portland, USA;
- 2 classes in Toluca, Mexico;
- Repetition of the experiment in the Montreal Anglophone school, with a different classroom group of students (Sept.-Dec. 2005).

Design of activities

The design of activities was developed taking into account the curriculum of Quebec (2004) for students in 10th grade. Eight activities were designed and tested in the classes as mentioned in the methodology. In what follows, we are centered on the activity 6.

The activity 6, was related to the factorization of $x^n - 1$. This task is a classical activity in the mathematics curriculum in several countries. For example, Munier & Aldon (1996), made not one but three experiments (1990, 1991 & 1993) around the same activity with 11th grade students in France:

- Using paper and pencil (work in teams, 2 hours),
- Using DERIVE (they thought that technology will facilitate the task…)
- Using DERIVE in a large project (three months)

What is different in our approach?

The difference in the design of our activity in relation to the factorization of $x^n - 1$ (and in general with other activities designed) is the ***balance we believe it is important to provide to the paper and pencil activities and their relationship with the use of the calculator***. In our theoretical framework, precisely we ***distinguish whether one or other approach and the students' actions that relate these approaches***.

Notation used in our learning model

The notations we use to identify students' actions are the following (see Hitt & Kieran, 2009):

- Task with paper and pencil $\mathbf{TASK_{P\text{-}P}}$;
- Task with CAS $\mathbf{TASK_{CAS}}$;
- Technique involving paper and pencil $\mathbf{TECH_{P\text{-}P}}$;
- Technique involving CAS $\mathbf{TECH_{CAS}}$;
- Semiotic production with paper and pencil $\mathbf{PROD_{P\text{-}P}}$;
- Semiotic production with CAS $\mathbf{PROD_{CAS}}$;
- A P-P task provoking a semiotic production $\boldsymbol{TASK_{P\text{-}P}} \underset{TECH_{P-P}}{\longrightarrow} \boldsymbol{PROD_{P\text{-}P}}$ that depends on a technique $\mathbf{TECH_{P\text{-}P}}$.
- A CAS task provoking a semiotic production $\boldsymbol{TASK_{CAS}} \underset{TECH_{CAS}}{\longrightarrow} \boldsymbol{PROD_{CAS}}$ that depends on a technique $\mathbf{TECH_{CAS}}$.
- Construction of a theory related to a technique:
  - $\left(\boldsymbol{TASK_{P\text{-}P}} \underset{TECH_{P-P}}{\longrightarrow} \boldsymbol{PROD_{P\text{-}P}}\right)$ **----> $\mathbf{THEO_{P\text{-}P}}$** (the dotted arrow means an internal construction)
  - $\left(\boldsymbol{TASK_{CAS}} \underset{TECH_{CAS}}{\longrightarrow} \boldsymbol{PROD_{CAS}}\right)$ **----> $\mathbf{THEO_{CAS}}$** (the dotted arrow means an internal construction)
- Conversion between productions: $\mathbf{PROD_{P\text{-}P}} \rightarrow \mathbf{PROD_{CAS}}$ and vice versa $\mathbf{PROD_{CAS}} \rightarrow \mathbf{PROD_{P\text{-}P}}$.
- Possible articulation between techniques and construction of a theory:
  - $\begin{tikzcd} & \mathbf{THEO} \\ (\mathbf{TASK_{P\text{-}P}} \longrightarrow \mathbf{PROD_{P\text{-}P}}) \arrow[ur, leftrightarrow, dashed] & (\mathbf{TASK_{CAS}} \longrightarrow \mathbf{PROD_{CAS}}) \arrow[u, dashed] \end{tikzcd}$ (the dotted arrows mean an internal construction).

In Activity 6, the theoretical constructs we have designated as $\mathrm{THEO}_{\mathbf{n}}$, $\mathbf{n} \in \{1, 2, 3, 4\}$. The activity 6, was designed to be worked in stages, where it was implicitly

referred to the formation of conjectures, the use of counter examples to reject some conjectures, and argumentations processes (at this level students do not know what is a mathematical proof).

Stage 1. Remembering factors.

The intention of this phase was to promote remembering notable products and comparing of the results with the calculator.

Stage 2. $THEO_1$: Given the expression $(x^n - 1)$ for a specific ***n***, it is equivalent to:

$(x - 1)(x^{n-1} + x^{n-2} + \ldots + x + 1)$– this theory based on the telescoping technique.

In this phase, we expected students to construct a technique (telescoping technique)

Stage 3. $THEO_2$: The equivalence of "$PROD_{P\text{-}P}$ and $TECH_{P\text{-}P}$" with "$PROD_{CAS}$ and $TECH_{CAS}$"

Considering the work of Guin & Trouche and Tall mentioned in the introduction, we put special attention in the design of this phase, asking the students to conciliate their productions in paper and pencil using the technique learned in phase 2, with the results provided by the calculator. This point is very important. Mathematicians and teachers have developed insights and knowledge that permit them to predict the results they're getting by using technology. However, students who do not have these skills, they tend to believe blindly in the results provided by the calculator or the computer without properly assimilating them in a process of problem solving. A calculus of a limit as requested by Guin & Trouche, to a mathematician or a professor is easy work, either directly calculating it or doing a correct interpretation of the graphic representation. However, students have a hard time in the classroom to do that kind of work, because their teachers probably did not pay attention to develop this kind of skills necessary to identify meaningful units in Duval's (1995) sense, when converting from one representation to another.

The intention of this phase was to promote reconciliation processes between students' pencil and paper productions with the results provided by CAS.

Stage 4. $THEO_3$: The conjecture that $(x^n - 1)$ will have exactly two factors, $(x - 1)$ and $(x^{n-1} + x^{n-2} + \ldots + x + 1)$, when ***n*** is odd (a false conjecture).

When we asked students about the factorization of $x^n - 1$, $\boldsymbol{n} \in \{2, 3, 4, 5, 6\}$. Our but was to promote the false assumption that it would have exactly two factors $(x - 1)$ and $(x^{n-1} + x^{n-2} + \ldots + x + 1)$, when ***n*** is odd.

Stage 5. Involving conceptual change – disproving a conjecture and generating a new one.

$THEO_4$: Rejecting previous conjectures to produce a new one: $(x^n - 1)$ will have exactly two factors, $(x - 1)$ and $(x^{n-1} + x^{n-2} + \ldots + x + 1)$, when ***n*** is a prime number.

In this phase, students were asked to factoring $x^n - 1$, $\boldsymbol{n} \in \{7, 8, 9, 10, 11, 12, 13\}$. Our but was that students would realize that their initial guess was false using the

$x^9 - 1 = (x - 1)(x^2 - x + 1)(x^6 + x^3 + 1)$ as a counter-example. This would allow the production of a new conjecture that could be verified and partially validated using the calculator, that is $x^n - 1$, will have exactly two factors, $(x - 1)$ and $(x^{n-1} + x^{n-2} + \ldots + x + 1)$, when n is a prime number.

Stage 6. Coordinating theories or distinguishing among theories

At this stage we asked the students the calculation of factorizations with ***n*** a very large number, so that if previous results applied correctly, they could obtain the correct results: Factoring $x^n - 1$, $\boldsymbol{n} \in \{2004, 3003, 853\}$.

Stage 7. Justifying conjecture -- or deepening of theory -- through proving

In this phase, students were asked to explain why $(x + 1)$ is always a factor of $x^n - 1$ for even values of n, $n \geq 2$. Because students do not know what is a mathematical proof, arguments were expected to convince their teammates about the generality of the result.

Students were interviewed, and about this activity, in a period of one hour and 10 minutes (see Hitt & Kieran, 2009) two boys working together came to discover the main point of the conjecture. In this interview there was no possibility of continuing with the last phase on argumentation and validation of the conjecture because it was to be developed in class with the whole group in the next lesson. In this same interview, if one of the students used the calculator, the other was writing the results, and the other way around. We can say that it was a coordination of actions when using the calculator. In the Eureka moment the students were using big numbers to test their expectations, that working exclusively in a paper and pencil environment is not possible to experiment with big numbers.

Discussion

Our design of ACODESA methodology (see Hitt, 2007, Hitt & Morasse, 2009, Hitt & Cortes, 2009) was implemented to promote learning in social interaction environments and use of technology. In Kieran's project, the design of activities was the principal but (see http://www.math.uqam.ca/APTE/Taches.html). In both projects, the intention was to provide the teacher with a framework to use in the mathematics classroom.

Our task design and analysis of the actions are important to take into account for future design and research. We believe that must be a balance in the activities of pencil and paper and technology.

With our design of activities and collaborative learning processes in contexts of social interaction, we have managed to generate in the classroom processes of conjecture, argumentation and processes of validation. These aspects we consider necessary to develop in the mathematics classroom before the formal process of mathematical proof.

We believe that a collaborative learning methodology and the ACODESA methodology and design activities that balance the pencil and paper production and use of technology would have a greater impact in the mathematics classroom. Necessary impact if we truly want to influence on the use of technology in the mathematics classroom.

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