Using Rational Arithmetic to Develop a Proof
"What Josef and Carl Saw"

Josef Boehm and Carl Leinbach

International DERIVE and CAS-TI User Group
Würmla, Austria
and
Gettysburg College
Gettysburg, Pennsylvania

nojoboehm@pgv.at
and
leinbach@gettysburg.edu

Lecture Proposal for the TI-Nspire & Derive Strand

ABSTRACT

It all began with an article in DNL #22 entitled **Finding a Limit via Geometric Reasoning** authored by Marvin Brubaker and Carl Leinbach. In that paper the limit of a recursively defined sequence of points was found by connecting successive points with straight lines, thus creating a nested sequence of triangles that seem to converge to the point (0.4, 0.4). While editing an archival edition of DNL #22, Josef correctly pointed out that the paper did not really have a proof of the limit, only a collection of heuristic evidence gained by zooming in on the suspected limit. He wrote to Carl asking if he had a mathematical proof that sequence did, in fact, converge to its claimed limit. Both Josef and Carl began independent work on the problem. Their initial step was the same. They each wrote a small DERIVE program to print out the first n terms of the sequence using the CAS's rational arithmetic display of the points. After this their two approaches differed.

In this presentation both Josef and Carl will discuss their approaches to constructing a proof that the sequence converges to its' claimed limit, thus supporting the visual evidence. They will also discuss the value of using the Rational Arithmetic to support the discovery of a strategy to accomplish their mathematical goal. If time permits, the presenters will investigate applying their approaches to other sequences of points.

Keywords
geometric intuition, rational Arithmetic, formal proof, limit, sequences