Dynamic Applets for Differential Equations and Dynamical Systems

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Workshop Proposal for the ACDCA strand

ABSTRACT

Dynamic/interactive graphing applets can be used to supplement standard computer algebra systems such as Maple, Mathematica, Derive, or TI calculators, in courses such as Calculus, Differential Equations, and Dynamical Systems. The addition of this type of software can lead to discovery learning, with students developing their own conjectures, and possibly proofs (as has happened when students explore geometry with interactive software). Also, topics which have traditionally been considered too advanced for standard undergraduate courses, such as the Poincare map in differential equations/dynamical systems, can be introduced at an intuitive level.

The presenter has both developed and used such applets in the teaching of various mathematics courses. Furthermore, in conjunction with several students, tools for easily creating new applets have been developed, so that mathematics educators can customize applets to their courses without having to become java programming experts (see the website created by these students at www.dynamicgrapher.com). In this workshop, participants will be lead through some guided investigations in differential equations/dynamical systems (described below) which can be used with students. Also, participants will be shown how to develop their own applets.

Possible investigations (depending on participant interest):

1. Interesting relationships between various numerical methods for solving the logistic differential equation $y' = ay(1 - y)$ are discovered. Euler's method, second-order Runge-Kutta (modified Euler) and fourth-order Runge-Kutta are compared as a parameter is smoothly changed in the differential equation. Computer algebra is used to confirm conjectures. This investigation was done by a student of the presenter and given as a talk by the student at professional meetings.
2. The equation for a damped pendulum $y'' + cy' + \sin(y)$ is investigated in the case of a large damping constant c. Varying the initial conditions interactively reveals a relationship between the solution curves in the phase plane and the $\sin(y)$ term in the differential equation. This investigation can then be extended to general equations of the form $y'' + cy' + f(y)$.

3. A form of the logistic differential equation with constant harvesting, given by $y' = ay(1-y) - d$, is investigated both interactively with an applet (the parameter d and the initial conditions are varied smoothly) and using computer algebra, in order to understand the bifurcation structure of the system as d is varied. In an extension, periodic harvesting is introduced via the model $y' = ay(1-y) - d(1+\cos(2\pi t))$. In order to locate periodic solutions, rather than constant ones as in the previous model, the Poincare map is introduced via an interactive applet, which is then used to compare the bifurcation structure of the two models.

Keywords

Interactive applets, computer algebra, differential equations, dynamical systems.