



Could it be possible to replace DERIVE with MAXIMA?

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TIME 2010

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CAS

- ◆ The Beginnings
- ◆ The software to be used
- ◆ Possibilities in Engineering
- ◆ DERIVE and MAXIMA

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DERIVE

- ◆ Very popular
- ◆ Associations
- ◆ Meetings
- ◆ Publications
- ◆ Author's experience
- ◆ Current situation

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MAXIMA and DERIVE

- ◆ Two CAS with similar features
- ◆ MAXIMA's characteristics
- ◆ The results will be shown

Use of CAS in Spanish Universities

- ◆ Inquiries sent to 158 Engineering Schools of 44 Spanish Universities
- ◆ 112 replies collected from 36 different Universities
- ◆ 44 centres that use DERIVE in 13 different Universities
- ◆ 20 centres, in 12 universities, using MAXIMA
- ◆ From DERIVE to MAXIMA in León, Murcia, Oviedo, UNED and UPM

MAXIMA versus DERIVE

- ◆ Step 1: Assessment of the general characteristics
- ◆ Step 2: Gauging the results of a real experiment
- ◆ Step 3: Implementation and analysis of several batteries of tests in Calculus
- ◆ Step 4: A quantitative evaluation

Essential requirements for math software

- ◆ Easy of use
- ◆ Symbolic, numerical and graphical capacities
- ◆ Availability of specific programming language
- ◆ Portability and interoperability
- ◆ Accessibility and easy installation
- ◆ Good maintenance
- ◆ Wide diffusion

General characteristics

- ◆ Some DERIVE advantages: Graphical interactivity and displays capacities (symbols and Greek letters)
- ◆ Some MAXIMA advantages: Open source and free software, more interactive with inputs, more natural programming language and easy to translate to other files

2. A real experience of change from DERIVE to MAXIMA

- ◆ 2009-2010 in UPM (Mathematical Analysis for first year students of Computer and Software Engineering)
- ◆ Laboratory sessions and Team work: Similar than done with DERIVE
- ◆ NO special time devoted to teach the tool
- ◆ Used for learning and evaluation

Some differences [Annex 1](#)

- ◆ Recursive sequences: MAXIMA more natural definition and more efficient evaluation
- ◆ Difference and Differential equations: Not normalization needed with MAXIMA
- ◆ Some MAXIMA outputs are unexpected
- ◆ DERIVE has the GEOMETRIC1 function for solving geometric recurrences

Student's Marks

- ◆ Similar
- ◆ DERIVE wins in marks
- ◆ MAXIMA wins in percentage

3. Systematic tests

DERIVE 6 and wxMaxima 5.20.1 (0.8.4)

- ◆ Defining and testing sixty typical features (see)

Successful results for both

- ◆ Solving with MAXIMA proposed problems of our book, Calculus I (real and complex numbers, elementary functions, limits and continuity, derivability, and integration)

Similar results than DERIVE

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Some Differences

- ◆ MAXIMA does not solve inequations neither equations with absolute values
- ◆ Complex numbers: Some additional use of options with MAXIMA
- ◆ Functions (more or less the same)
- ◆ MAXIMA does not compute limits and derivatives of piece-wise functions
- ◆ DERIVE offers better features for finding primitives

Annex 2

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4. Quantitative evaluation

ISO 9126 standar

- ◆ Functionality
- ◆ Reliability
- ◆ Usability
- ◆ Efficiency
- ◆ Maintainability
- ◆ Portability

Quantitative evaluation

Designed for a Calculus I course

- ◆ Reliability **Ok**
- ◆ Efficiency **Ok**

Quantitative evaluation

- ◆ Functionality (40%)
- ◆ Usability (40%)
- ◆ Maintainability (10%)
- ◆ Portability (10%)

Functionality: The marks for each of sixty metrics

- ◆ It does not work (1)
- ◆ It works, but the result is unsatisfactory (2)
- ◆ In general it works, although in some foreseeable cases it does not (3)
- ◆ It works well, although in some cases it takes longer than desirable or the output is hard to handle (4)
- ◆ It works efficiently and the output is the expected one (5)

Functionality: General Results

- ◆ Detailed results of the assessment of both CAS can be seen in Annex 3
- ◆ Mean values: **DERIVE:4.73, MAXIMA:4.17**
- ◆ Both MAXIMA and DERIVE meet the requisite of functionality, consisting of that **75% of the metrics evaluated have a value equal to or greater than 4**, which means that both can be used as a support tool in a Calculus course for Engineers

Usability: Subcategories evaluation

Variable	DERIVE	MAXIMA
Accessibility and easy installation	3	5
Learning time	5	5
Accessible documentation	4	4
Friendly interface	4	4
Graphical interface	5	4
Presentation on screen	5	4
Help systems	5	3
Mean value	4.43	4.14

Evaluation of Usability for each metric

- ◆ It is not intuitive and I have not found it on the help page (1)
- ◆ It is not very intuitive and the help offered is not very explicit (2)
- ◆ The command is in the menu and/or the help page, but the help is not sufficiently clear (3)
- ◆ The way is intuitive or there is a command key on the menu but I need the help page (4)
- ◆ It has been easy for me to do it and the output is easy to handle (5)

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Quantitative evaluation: Usability

	DERIVE	MAXIMA
GENERAL	4.43	4.14
CALCULUS METRICS	4.6	4.26
MEAN VALUES	4.51	4.20

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Quantitative evaluation: Global results

Category	DERIVE	MAXIMA
Functionality (40%)	4.73	4.17
Usability (40%)	4.51	4.20
Portability (10%)	3	5
Maintainability (10%)	1	4
Final score	4.096	4.24

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Conclusions

- ◆ YES
- ◆ Peculiarities

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Final remark

◆ *DERIVE's friends*



◆ *MAXIMA's users*



THANK YOU

GRACIAS



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Annex 1. Sequences with Maxima

RECURRENT SEQUENCES

(%i1) `a[1]:=1;`

(%o1) 1

(%i2) `a[n]:=n+a[n-1];`

(%o2) $a_n := n + a_{n-1}$

(%i3) `a[5];`

(%o3) 15

Te definition of a recurrent sequence with MAXIMA is very easy.

But it is necessary to unassign the value of a, before introducing a new definition for a[n].

(%i4) `kill(a);`

(%o4) *done*

Fibonacci Sequence

(%i5) `a[0]:=0;`

(%o5) 0

(%i6) `a[1]:=1;`

(%o6) 1

(%i7) `a[n]:=a[n-1]+a[n-2];`

(%o7) $a_n := a_{n-1} + a_{n-2}$

(%i8) `a[200];`

(%o8) 280571172992510140037611932413038677189525

Evaluation of recursive sequences with Maxima is more efficient than DERIVE recursive evaluation

(%i9) `time(%);`

(%o9) [0.04]

DIFFERENCE EQUATIONS

(%i10) `load(solve_rec);`

(%o10)

C:/ARCHIV~1/MAXIMA~2.1/share/maxima/5.20.1/share/contrib/solve_rec/solve_rec

A general command for which normalization is not necessary

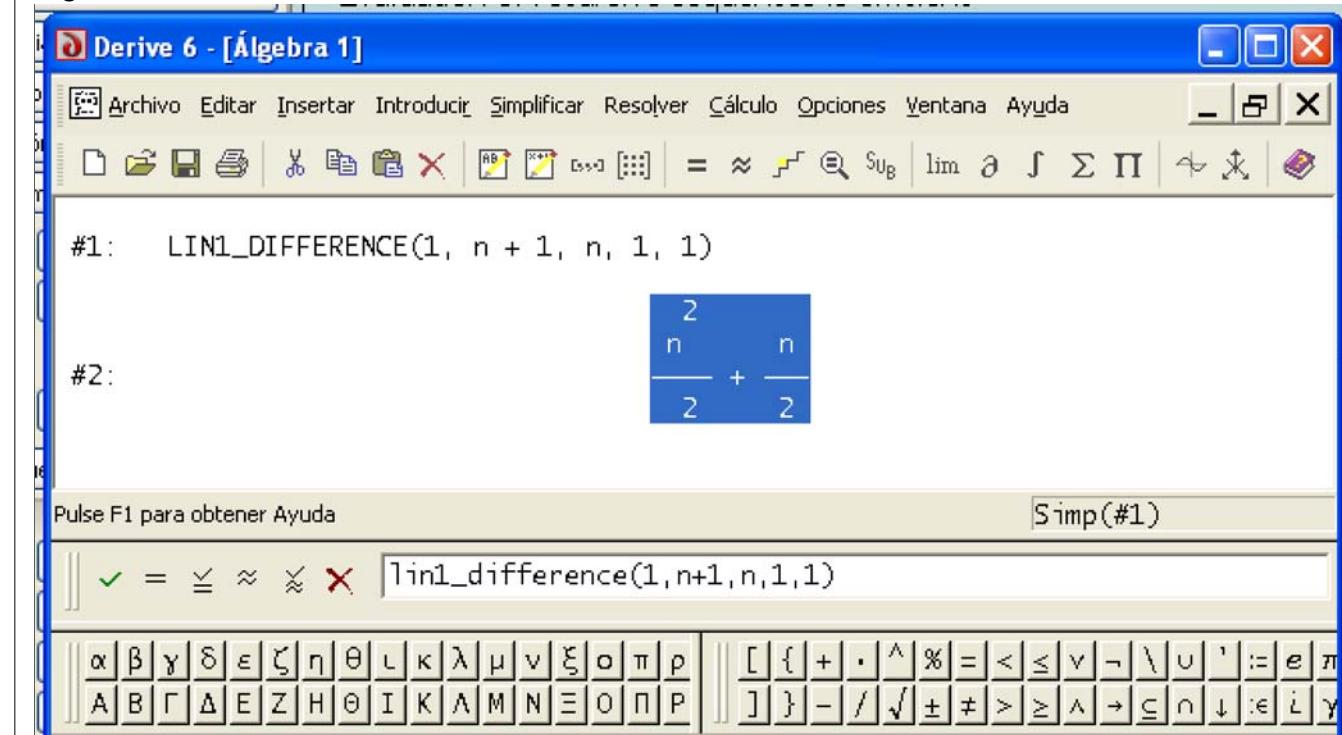
```

(%i11) solve_rec(x[n]=x[n-1]+n,x[n],x[1]=1);
(%o11) x_n =  $\frac{(n - 1)(n + 2)}{2} + 1$ 

```

```
(%i12) expand(x[n]=((n-1)*(n+2))/2+1);
(%o12) x_n =  $\frac{n^2}{2} + \frac{n}{2}$ 
```

Figura 1: SOLUTION WITH DERIVE



↳ Sometimes the output is unexpected

```
(%i13) solve_rec(x[n+1]=(n)*x[n]+(n+1)!,x[n],x[1]=2);
(%o13) x_n =  $\frac{(n^2 + n + 2 \%r2)(n + 1)!}{2 n (n + 1)} - (\%r2 + 1)(n - 1)! + 2(n - 1)!$ 
```

```
(%i14) subst(0, %r2, %);
(%o14) x_n =  $\frac{(n^2 + n)(n + 1)!}{2^n(n + 1)}$  + (n - 1)!
```

Figura 2: SOLUTION WITH DERIVE

The screenshot shows the Derive 6 software interface. The menu bar includes Archivo, Editar, Insertar, Introducir, Simplificar, Resolver, Cálculo, Opciones, Ventana, and Ayuda. The toolbar contains various icons for file operations and mathematical functions. The main workspace displays the following steps:

```

#3: LIN1_DIFFERENCE(n, (n + 1)!, n, 1, 2)
#4: 
$$\left( \frac{n^2}{2} + \frac{n}{2} + 1 \right) \cdot (n - 1)!$$


```

A status bar at the bottom indicates "Simp(#3)". Below the workspace is a keyboard-like input field with Greek letters (α, β, γ, δ, ε, ζ, η, θ, κ, λ, μ, ν, ξ, ο, π, ρ), symbols ([, {, +, ., ^, %, =, <, ≤, v, ~, \, u, ', :=, e, π,], }, -, /, √, ±, ≠, >, ≥, ∧, →, ⊆, ⊂, ↓, :e, i, γ), and a numeric keypad.

Second order linear equations

(%i15) `solve_rec(x[n+2]+5*x[n+1]+6*x[n]=0,x[n],x[1]=1,x[2]=4);`
 (%o15) $x_n = 2(-3)^n + 7(-2)^{n-1}$

(%i16) `solve_rec(x[n+2]+4*x[n+1]+4*x[n]=0,x[n],x[1]=1,x[2]=4);`
 (%o16) $x_n = \left(\frac{3n}{2} - 2\right)(-2)^n$

Figura 3: WITH DERIVE

The screenshot shows the Derive 6 software window with the title "Derive 6 - [Álgebra 1]". The menu bar includes Archivo, Editar, Insertar, Introducir, Simplificar, Resolver, Cálculo, Opciones, Ventana, Ayuda. The toolbar contains various mathematical symbols and functions. The input field contains the following sequence of commands and results:

```

#5: LIN2_CCF_BV(5, 6, 0, n, 1, 1, 2, 4)
#6: 2·3n ·COS(π·n) - 7·2n - 1 ·COS(π·n)
#7: LIN2_CCF_BV(4, 4, 0, n, 1, 1, 2, 4)
#8: 2n - 1 ·(3·n - 4) ·(-1)n

```

The status bar at the bottom shows "simp(%r)".

Second order linear equations with complex roots on the characteristic polynomial

(%i17) `solve_rec(x[n+2]+x[n]=0, x[n], x[1]=1 , x[2]=4);`

(%o17)
$$x_n = \frac{(\%i - 4)(-\%i)^n}{2} - \frac{(\%i + 4)(-1)^{n/2}}{2}$$

x[n] is a sequence of real numbers

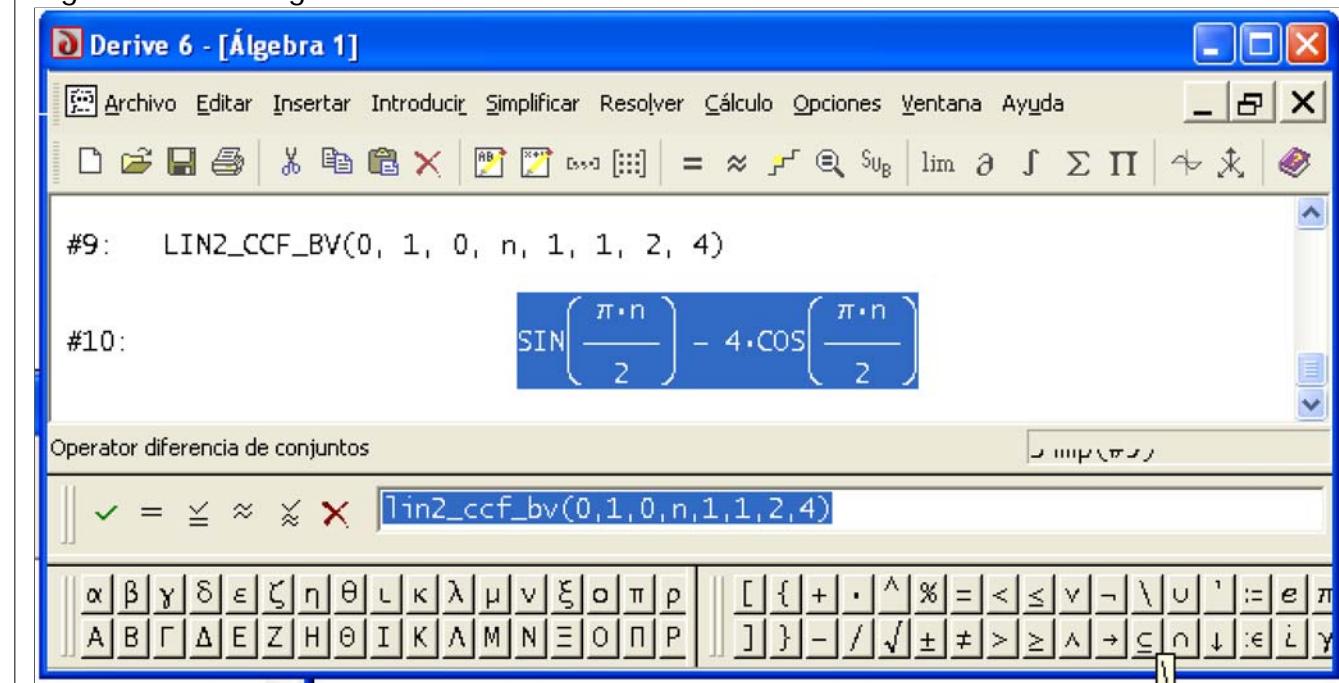
(%i18) `x[n]:=((%i-4)*(-%i)^n)/2-((%i+4)*(-1)^(n/2))/2;`

(%o18)
$$x_n := \frac{(\%i - 4)(-\%i)^n}{2} - \frac{(\%i + 4)(-1)^{n/2}}{2}$$

(%i19) `makelist(rectform(x[n]),n,1,20);`

(%o19) `[1, 4, -1, -4, 1, 4, -1, -4, 1, 4, -1, -4, 1, 4, -1, -4, 1, 4, -1, -4]`

Figura 4: DERIVE gives the real solution



A Nonhomogeneous second order linear equations, that DERIVE does not solve

(%i20) kill(x);
(%o20) done

(%i21) solve_rec(x[n+2]+5*x[n+1]+6*x[n]=1,x[n],x[1]=1,x[2]=4);
(%o21) $x_n = \frac{5(-2)^{n+1}}{3} - \frac{23(-3)^{n-1}}{4} + \frac{1}{12}$

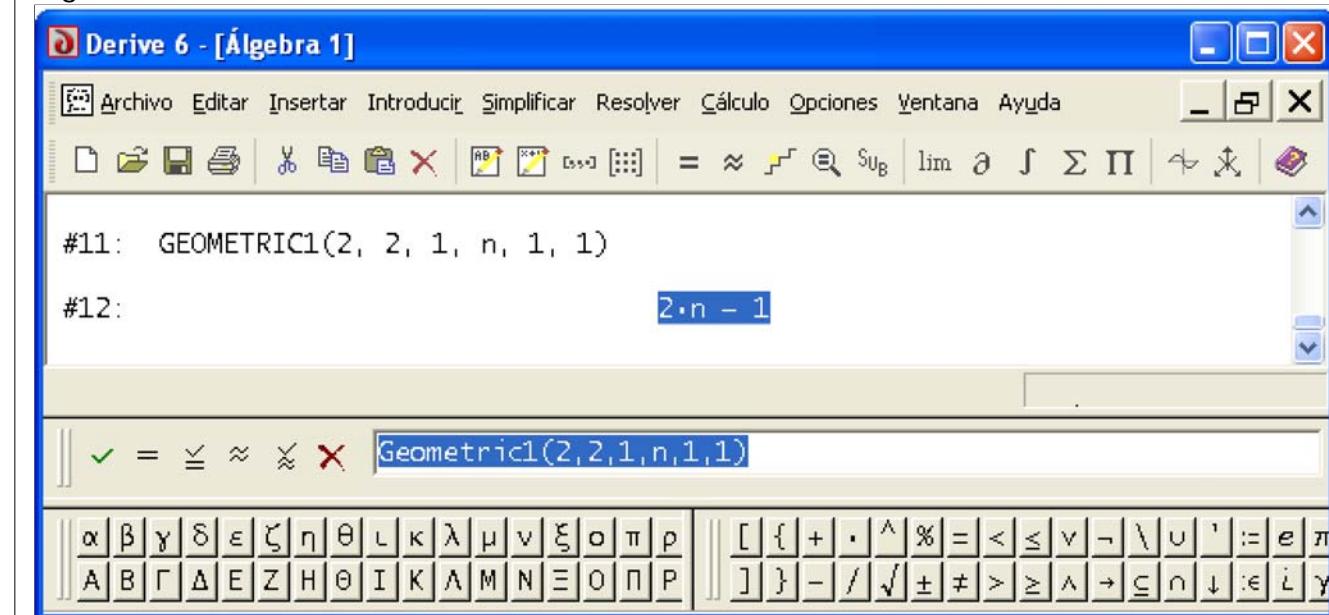
(%i22) solve_rec(x[n+2]+5*x[n+1]+6*x[n]=n,x[n],x[1]=1,x[2]=4);
(%o22) $x_n = \frac{31(-3)^n}{16} + \frac{61(-2)^{n-1}}{9} + \frac{n}{12} - \frac{7}{144}$

A Geometric recurrence tha Maxima does not solve

(%i23) kill(all);
(%o0) done

(%i1) solve_rec(x[2*n]=2*x[n]+1,x[n],x[1]=1);
(%o1) solve_rec(x[2*n]=2*x[n]+1,x[n],x[1]=1)

Figura 5: Solution with GEOMETRIC1 command



Sum of power series

(%i2) $x^n/n!$
 (%o2) $\frac{x^n}{n!}$

(%i3) $\text{sum}(x^n/n!, n, 0, \text{inf}), \text{simpsum};$
 (%o3)
$$\sum_{n=0}^{\infty} \frac{x^n}{n!}$$

DERRIVE simplifies this sum to $\exp(x)$, but do not get the power series of $\exp(x)$

(%i4) $\text{powerseries}(\exp(x), x, 0);$
 (%o4)
$$\sum_{i1=0}^{\infty} \frac{x^{i1}}{i1!}$$



ANNEX 2: Unexpected results with Maxima

Problems by simplifying an absolute value

--> $a(n):=(-1)^n 3^n / (4^n);$

$$(\%o17) \quad a(n) := \frac{(-1)^n 3^n}{4^n}$$

--> $\text{abs}(a(n)^{(1/n)});$

$$(\%o18) \quad -\frac{3((-1)^n)^{1/n}}{4}$$

--> $\text{limit}(\%, n, \text{inf});$

$$(\%o19) \quad \frac{3}{4}$$

--> $\text{abs}(a(n))^{(1/n)};$

$$(\%o20) \quad \frac{3|(-1)^n|^{1/n}}{4}$$

--> $\text{limit}(\%, n, \text{inf});$

$$(\%o21) \quad -\frac{3}{4}$$

Figura 1: DERIVE simplifies to $\frac{3}{4}$

The screenshot shows the Derive 6 software window with a menu bar and toolbar. The main workspace contains the following input and output:

```

#1: a(n) := 
$$\frac{(-1)^n \cdot 3^n}{n^4}$$

#2: 
$$\left[ |a(n)|^{1/n}, \left| a(n) \right|^{1/n} \right]$$

#3: 
$$\left[ \frac{3}{4}, \frac{3}{4} \right]$$

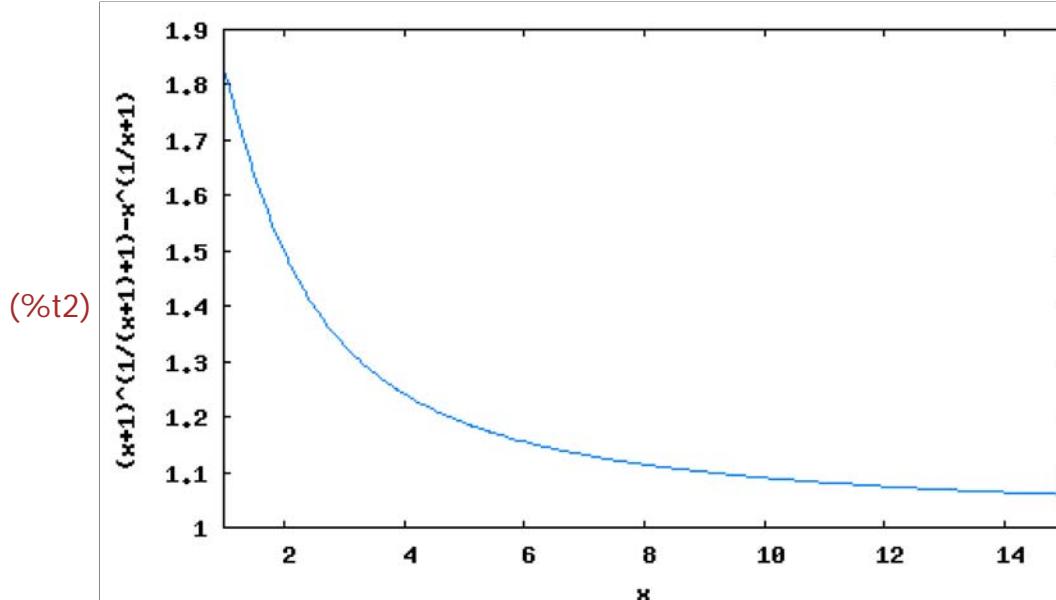

```

The status bar at the bottom left says "Pulse F1 para obtener Ayuda" and the right says "Simp(#2)". The keyboard area shows Greek letters and symbols.

A mistake in calculating a limit

```
--> f(x):=x^(1+1/x);
(%o1) f(x):= $x^{1+\frac{1}{x}}$ 
```

```
--> wxplot2d([f(x+1)-f(x)], [x,1,15])$
```



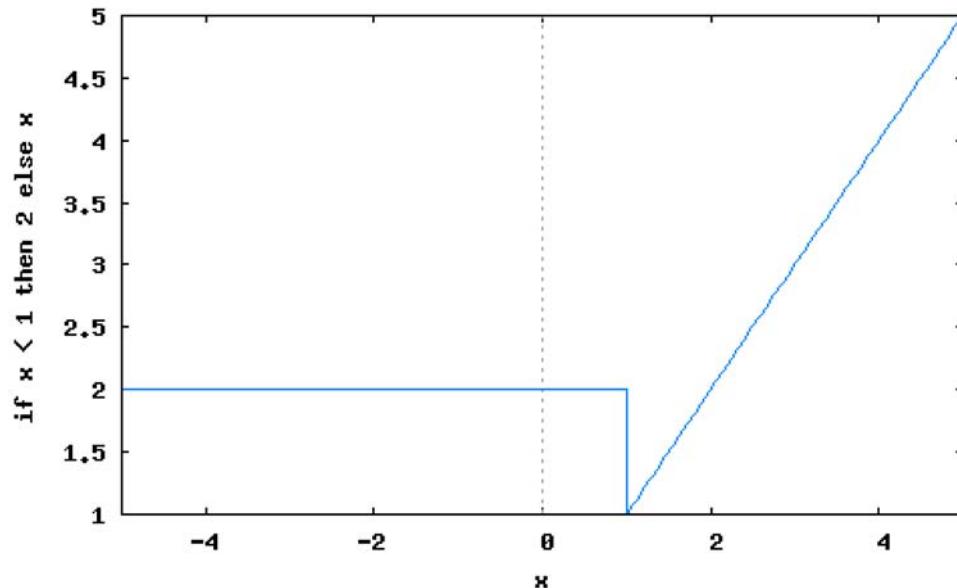
```
--> limit(f(x+1)-f(x), x, inf);  
(%o3) inf
```

DERIVE does not simplify this limit. The correct result is 1.

A piece-wise function, defined with if

```
--> f(x):= if x<1 then 2 else x;  
(%o4) f(x):= if x < 1 then 2 else x
```

```
--> wxplot2d([f(x)], [x,-5,5])$
```



```
--> limit(f(x), x, 1, minus);  
(%o8)           lim      x-> 1 -  
               if x < 1 then 2 else x
```

```
--> limit(f(x), x, 0);  
(%o9)           lim      x-> 0  
               if x < 1 then 2 else x
```

Derive simplifies correctly these limits and the following derivative

```
--> diff(f(x),x);  
(%o10) (if x < 1 then 2 else x)  $\frac{d}{dx}$ 
```

Figura 2: A piece-wise function, defined with IF

The screenshot shows the Derive 6 software interface with the file **[Algebra 1 annex2.dfw]** open. The menu bar includes Archivo, Editar, Insertar, Introducir, Simplificar, Resolver, Cálculo, Opciones, Ventana, and Ayuda. The toolbar contains various mathematical symbols and functions. The input window displays the following code:

```

f(x) :=  

  If x < 1  

#1:      2  

         x  

#2: [lim f(x), lim f(x), lim f(x)]  

     x->1+   x->1-   x->1  

#3: [1, 2, 1]  

#4:  $\frac{d}{dx} f(x)$   

#5: IF(x < 1, 0, 1)  

#6: f'(1)  

#7:  $\boxed{\quad}$   


```

The status bar at the bottom shows the path **C:\Documents and Settings\garcial\Mis documentos\educativos\TIME2010\Calculo\annex2.dfw guard** and the command **Simp(#6)**. The keyboard is visible at the bottom.

□

□ Integrals which Maxima does not calculate, but Derive do it

□

```
--> integrate(abs(x),x,-1,1);  

(%o14) 
$$\int_{-1}^1 |x| dx$$

```

□

```
--> integrate(1/(sqrt(x^2+1)+1),x);  

(%o15) 
$$\int \frac{1}{\sqrt{x^2+1} + 1} dx$$

```

Figura 3: Integrals

Derive 6 - [Álgebra 1 annex2.dfw]

Archivo Editar Insertar Introducir Simplificar Resolver Cálculo Opciones Ventana Ayuda

#8: $\int_{-1}^1 |x| dx$

#9: 1

#10: $\int \frac{1}{\sqrt{x^2 + 1} + 1} dx$

#11: $\text{LN}(\sqrt{x^2 + 1} + x) - \frac{\sqrt{x^2 + 1} - 1}{x}$

int(1/(sqrt(x^2+1)+1),x)

Keyboard:

α	β	γ	δ	ε	ζ	η	θ	ι	κ	λ	μ	ν	ξ	ο	π	ρ	σ	τ
Α	Β	Γ	Δ	Ε	Ζ	Η	Θ	Ι	Κ	Λ	Μ	Ν	Ξ	Ο	Ρ	Σ	Τ	

[{	+	*	^	%	=	<	<=	>	>=	-	\	U	'	:	e	π	∞	∂
]	}	-	/	√	±	≠	>	≥	∧	→	≤	n	↓	ε	i	γ	°		

EVALUACIÓN DE METRICAS FUNCIONALIDAD Y USABILIDAD

	Derive F	Derive U	Maxima F	Maxima U
1.1. Aritmética racional exacta	5	5	5	4
1.2. Definición y almacenamiento de variables para cálculos intermedios.	5	5	5	4
1.3. Números reales (aritmética exacta y aproximada de precisión arbitraria)	5	5	5	4
1.4. Aritmética entera (factorización, cociente y resto de la división entera , mcd...)	5	5	5	5
1.5. Operaciones con números complejos	5	5	5	5
1.6. Módulo y argumento de números complejos	5	5	5	5
1.7. Funciones exponenciales y logarítmicas.	5	4	4	4
1.8. Funciones trigonométricas.	5	4	5	4
2.1. Manejo de expresiones algebraicas (incluyendo descomposición en fracciones simples)	5	4	4	5
2.2. Factorización de polinomios.	5	5	5	5
2.3. Resolución de ecuaciones polinómicas (soluciones exactas y/o aproximadas).	5	5	4	4
2.4. Resolución de ecuaciones trigonométricas.	4	4	4	4
2.5. Resolución de ecuaciones exponenciales y logarítmicas.	4	4	3	4
2.6. Resolución de ecuaciones con valores absolutos.	5	5	1	
2.7. Resolución de inecuaciones.	5	5	1	
2.8. Resolución de sistemas sencillos (lineales o no lineales).	5	5	5	5
3.1. Programación de algoritmos sencillos (unas pocas instrucciones consecutivas).	4	4	5	5
3.2. Definir y modificar el dominio de una variable (uso de esta información)	4	4	4	4
4.1. Definición y evaluación de funciones reales de variable real.	5	5	5	5
4.2. Realización de tablas de valores de funciones.	5	5	5	5
4.3. Curvas explícitas (gráfica de $f(x)$) (traza, cambio de escala..)	5	5	5	3
4.4. Curvas implícitas	5	5	4	4
4.5. Curvas en paramétricas.	5	5	4	4
4.6. Representación de una lista de puntos	5	5	5	4
4.7. Gráficas simultáneas y animaciones	5	5	4	4
4.8 Cálculo de límites y límites laterales.	5	5	5	5
4.9. Límites en funciones definidas en intervalos	4	5	3	5
4.10. Cálculo de función derivada y derivada en un punto.	5	5	5	4
4.11. Derivada de funciones con valores absolutos o definidas a trozos.	4	5	2	2
4.12. Facilidad para probar por inducción la expresión para la derivada n-ésima	4	5	4	4
4.13. Programación intuitiva de la obtención de la tangente o del Método de Newton.	5	5	5	5
4.14. Polinomio de Taylor (facilidad del cálculo y el uso para aproximaciones)	5	5	5	4

4.15. Interpoalación.	4	5	5	4
4.16. Cálculo de primitivas.	5	5	3	5
4.17. Funciones de apoyo del cálculo de primitivas.	5	5	4	4
4.18. Manejo de funciones definidas por integrales. (evaluación aproximada, representación gráfica, derivación).	5	5	3	5
4.19. Integrales impropias.	5	5	5	5
4.20 Integrales especiales, f. elípticas Ganma y Beta.	5	5	5	5
4.21. Programación de métodos numéricos: Trapecio y Simpson compuesto.	5	4	5	5
5.1. Definición y evaluación eficiente de sucesiones recursivas.	4	4	5	4
5.2. Cálculo de límites de sucesiones.	4	5	3	5
5.3. Ecuaciones en diferencias lineales de primer orden.	5	3	4	5
5.4. Ecuaciones en diferencias lineales de segundo orden.	5	3	4	5
5.5. Ecuaciones en diferencias geométricas.	4	3	1	
5.6. Simplificación de algunas sumas de n términos.	5	5	3	5
5.7. Suma de series geométricas.	5	5	5	5
5.8. Suma de series aritmético-geométricas y telecopicas	5	5	1	
5.9. Suma aproximada de series	5	5	5	4
5.10. Aplicación de modo efectivo criterios como la raíz o el cociente.	5	5	3	5
5.11. Series de funciones (de Taylor y de Fourier).	3	3	5	3
6.1. Gráficas de superficies. (gráficas simultáneas, curvas de nivel, ...).	4	4	5	3
6.2. Curvas en el espacio.	4	4	5	3
6.3. Coordenadas, polares, cilíndricas y esféricas	4	4		1
6.4. Derivadas parciales	5	5	5	5
6.5. Gradiente	5	5	5	4
6.6. Hessiano	5	4	5	5
6.7. Jacobiano	5	5	5	5
6.8. Operadores vectoriales (rotacional, divergencia).	5	4	5	4
7.1. Resolución de EDO's lineales de primer orden.	5	4	5	5
7.2. Resolución de EDO's lineales de segundo orden.	5	4	5	5
VALORES MEDIOS	4,73333333	4,6	4,16666667	4,26315789