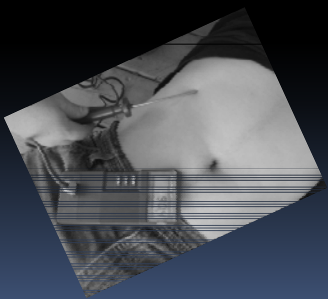
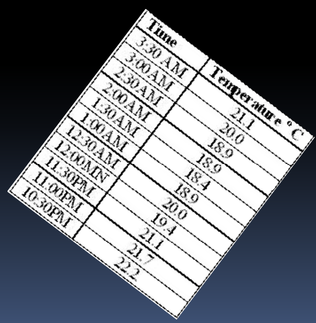
MATHEMATICAL MODELS FOR ESTIMATING TIME SINCE DEATH DURING THE EARLY POSTMORTEM PERIOD



L. Carl Leinbach
Gettysburg College, USA
leinbach@gettysburg.edu



Time	Temperature °C
3:30 AM	21.1
3:00AM	20.0
2:30AM	18.9
2:00AM	18.9
1:30AM	18.4
1:00AM	18.9
12:30AM	20.0
12:00MN	19.4
11:30PM	21.1
11:00PM	21.7
10:30PM	22.2

Who Is The Client?

Determining the time since death is a major responsibility of Forensic Investigators. This paper looks at several models that have been constructed for making this determination.

In the Field:
- Coroners
- Police
- Crime Scene Specialists

In the Lab:
- Forensic Pathologists
- Prosecutors
- Attorneys
- Insurance Companies

Immediate Family (Generally Notified by the Coroner)

What is the Definition Of Forensics?

Relating to, used in, or appropriate for courts of law or for public discussion or argumentation

NOTE: The word goes back to Ancient Rome where complaints, both criminal and civil, were brought before the Senate in the forum. Generally, the party that made the most convincing argument won.

Difference Between A Medical Autopsy And A Forensic Autopsy

- A medical autopsy is to establish the cause of death
- A forensic autopsy is to not only establish the cause of death, but also to collect evidence on, or about, or within the victim's body that may be used in a court of law.

Who Is The Coroner?

- Elected or appointed to office
- Second oldest row office in the United States (second only to the sheriff)
- In some states or jurisdictions the office may be called a 'Medical Examiner'
- Backgrounds of the person serving as coroner may vary

What Are The Duties Of The Coroner?

1. Establish cause and manner of death
2. Identification of the deceased
3. Secure the belongings on or about the deceased
4. Notify the next of kin of the deceased
5. Establish the time of death

Why Is It Important to Establish Time Of Death?

• In a criminal death, it is one way to help rule suspects in or out
• For some insurance claims it can be used to tie in cause with the time frame of an illness or injury.
• It may be years to determine the causative agent or person.

Is Mathematics The Only Way? – No!

1. Witnesses to the victim expiring
2. Phone company or hotel phone records. Victim was on the phone or died while using the phone.
3. Stopped watch on victim of an accidental death
4. Death within a medical facility
5. Specific forensic investigations: forensic odentalogy, forensic entomology, forensic anthropology, forensic pharmacology, forensic pathology

Temperature Based Models

• Data is relatively easy to obtain on scene.
• Generally assume stable ambient temperature
• Have received most attention by forensic practitioners
• Generally Dependant on Patient Size, Body Position, Body Characteristics and Time of Discovery
• Most Common Sites for Collecting Data: Rectum or Liver

Observation of Autolytic Processes (Autolytic: breakdown of cell of tissue)

• Rigor mortis: temporary rigidity of muscles after death, at best can give only a rough estimate fo between 4 and 36 hours
• Observation of various organs: gall bladder, esophagus, stomach, duodenum, etc.
• Analysis of concentration of Potassium ions in the vitreous humor in the eye – most be done within approximately 9 hours postmortem
• Generally these methods require a laboratory environment and are not used on the scene

Earliest Known Temperature Based Model

Bodies will cool at a rate of 1° C per hour

Advantages:

- Requires only one temperature measurement
- Can easily be applied on the scene with one external measurement of the body temperature taken at virtually any site.
- Calculation is extremely simple and can be done by a lay person

Disadvantages

- Generally inaccurate (especially in an extended time frame)
- Ignores laws for cooling bodies, experimental data, and, of course, thermodynamics

Use of this model began in the mid Nineteenth Century and is still used to this day.

Single Exponential Model

Based on Newton's "Law of Cooling" first published in 1701 as a paper entitled "Scala Graduum Caloris" that appeared in the Philosophical Transactions of the Royal Society of London.

While testing the cooling of a red hot iron bar Newton concluded:

> "The excess of degrees of heat [temperatures] of the iron ... were in geometrical progression, when the times were in arithmetical progression."

Stating this in more modern language:

> *The rate of cooling is linear in the difference between the temperature of the heated object and the surrounding (ambient) temperature.*

In 1868, H. Rainy applied this rule to the cooling of deceased bodies in an article in the Glasgow Medical Journal entitled "On the cooling of Dead Bodies as Indicating of the Length of Time Since Death."

Rainy’s approach was the first method for estimating time since death that took into account a scientific principle for estimating the manner in which bodies cool.

Consider the following example:

Time	Liver Temperature °C
3:30 AM	34.00
3:45 AM	33.89
4:00 AM	33.67
4:15 AM	33.56
4:30 AM	33.33
4:45 AM	33.00
5:00 AM	32.78

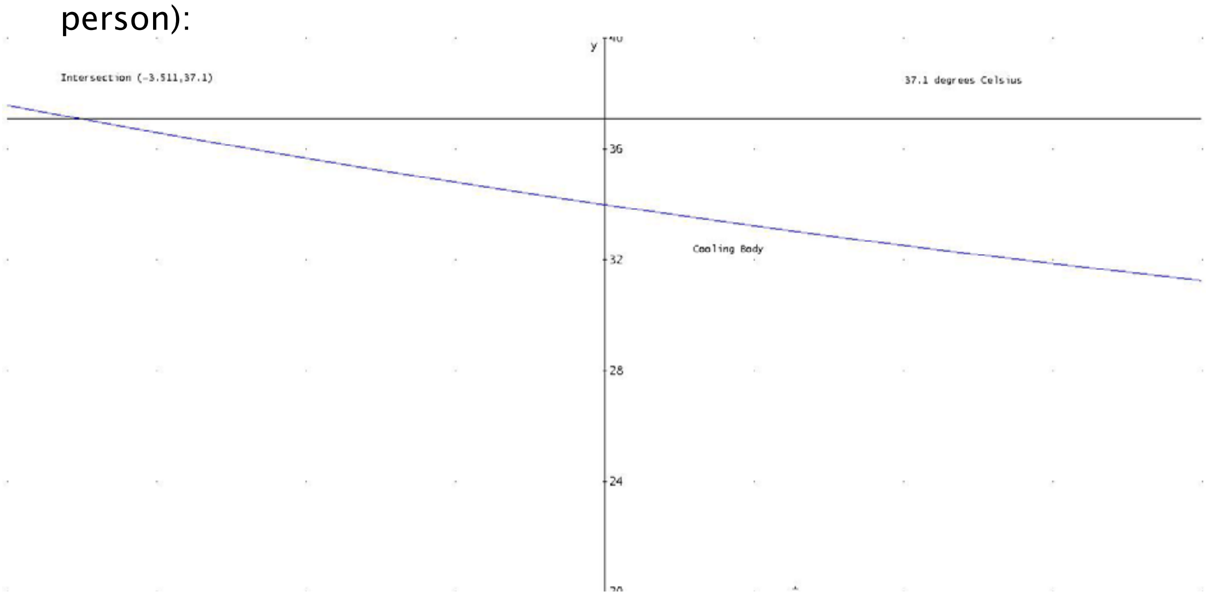
Liver Temperatures Taken At The Scene Of Death

Subtracting the 3:30AM ambient temperature of 21.1° Celsius and using the Diffenential Equation Solver in DERIVE and applying it to the equation resulting from Newton’s Law of Cooling, we obtain

$$\text{diff}(t) := 11.9 \cdot e^{-0.06585623942 \cdot t}$$

Adding the ambient temperature to diff(t), we have an equation for the body temperature at time, t based on Newton’s Law of Cooling.

Drawing the graph of the calculated body temperature and the constant 37.1 (the assumed normal body temperature for a live person):



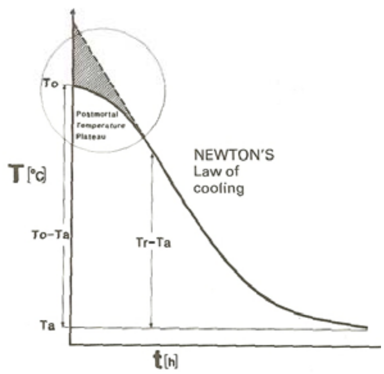
We see that the estimated time since death is approximately 3.5 hours earlier. So, the approximate time of death is 12 midnight.

Rainy's Method
Based on Newton's Law of Cooling

Advantages
- Uses a proven scientific rule for modeling the cooling process
- Gives results that are good approximations based on scientific experimentation
- Data can be gathered on the death scene
- Confidence interval on time of death fulfills the requirements for forensic testimony in a court of law

Disadvantages
- Requires several temperature measurements taken *at least* fifteen minutes apart – preferably longer time intervals, some texts advise one hour apart
- Assumes a constant ambient temperature – seldom found at the scene
- Victim's body size, clothing, and positioning (sitting, lying on floor or in bed and covered)
- Rainy observed that during the first three to four hours the cooling may be at a slower rate than after that time period

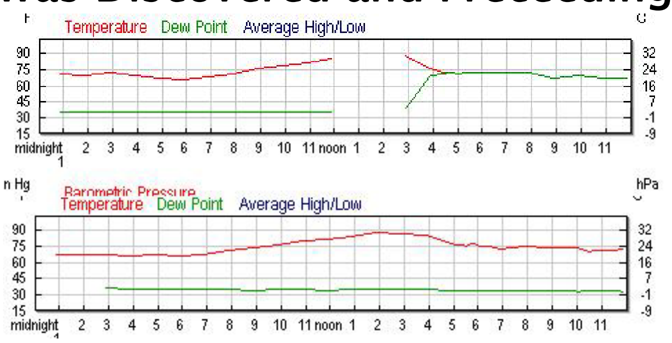
Graph Of Rainy's Experimental Data
And
Newton's Rule For Cooling Bodies



From: <u>The Estimation of the Time Since Death in the Early Postmotem Period</u>, Bernard Knight, editor, ©1995 Edward Arnold, p 56

We will return to this later, but first the ambient temperature issue

Temperature History From The Day Body Was Discovered and Preceeding Day



Graphs from "Weather History" link at URL http://www.wunderground.com

The ambient temperature, like that at most death scenes, is obviously not a constant. Also, like most real world data, it does not appear to resemble any closed form function that we recognize.

We can, however, take readings from the weather data and define a linear interpolation function in Derive to approximate the data given in the graphs. NOTE: We are not restricted to linear interpolation. We could use some other interpolating scheme.

Derive Function for Linear Interpolation

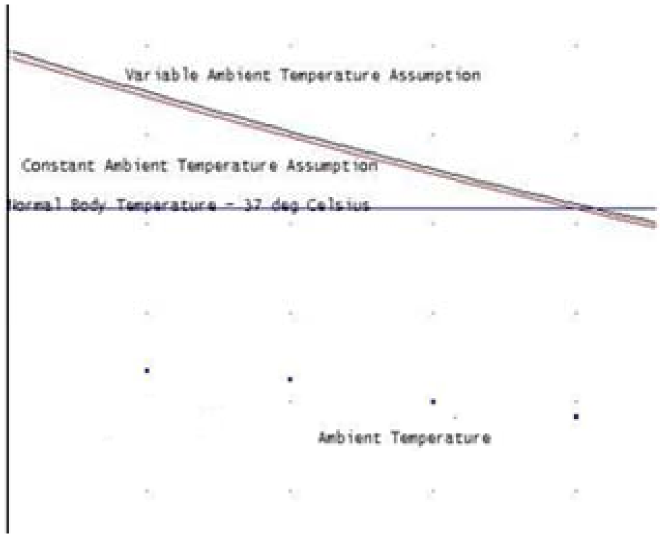
```
K(t, tim, amb, i, n) ≔
  Prog
    n ≔ DIM(tim)
    i ≔ 1
    Loop
      If t = tim↓1
         RETURN amb↓1
      If t > tim↓i ∧ t < tim↓(i + 1)
         RETURN amb↓i + (amb↓(i + 1) - amb↓i)/(tim↓(i + 1) -tim↓i)·(t - tim↓i)
      i ≔ i + 1
      If i > n - 1
         RETURN 0
```

The next step is to replace the constant ambient temperature, K, in Newton's Law of Cooling with the above function and apply a numerical DE Solver, such as Fourth Order Runge-Kutta to compute the earlier body temperatures back to the point where it is 37.1° Celsius.

RK([- 0.06177·(y - K(t, TOD, AMB))], [t, y], [1.5, 22.06], -0.1, 150)

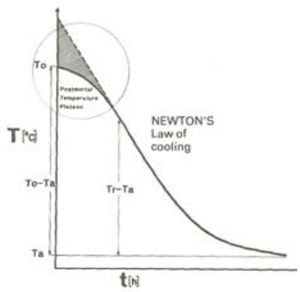
TOD and AMB are vectors for the time of day and ambient temperature, respectively

Graphs Of Rainy's Method Using Constant and Variable Temperatures



While there is some difference in the results, the estimation of the time of death is not significant in this case. NOTE: The ambient temperature data is plotted in intervals of one hour. In this case the difference in the estimates was less than 10 minutes.

Back to Rainy's Plateau



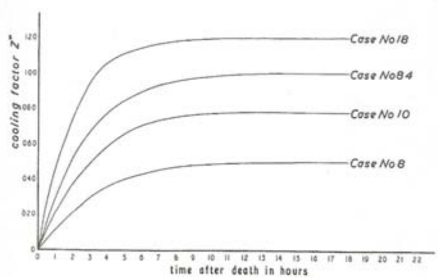
The fact that the rate of cooling was not really constant puzzled researchers for nearly 100 years after H. Rainy adapted Newton's Law of Cooling in 1868.

In 1962 T. K. Marshall (an M.D.) and F. E. Hoare (a Physicist) published "Estimating the time of death. The rectal cooling after death and its mathematical expression." and two follow up papers in the *Journal of Forensic Science* that explored the phenomena and developed a model for the sigmoidal shaped curve.

The Research of Marshall and Hoare

Early in their first paper, Marshall and Hoare established the fact that the basic assumption of Newton's Law of Cooling was invalid when applied to the very early stages of the cooling of a deceased human body.

Newton's Law of Cooling: *If $\theta(t)$ denotes the difference in the body temperature and ambient temperature, and $Z^*(t)$ denotes the rate of change of* $\theta(t)$ *then $Z^*(t) = K$, a constant.*



These graphs are typical of Marshall and Hoare's study of 100 deceased

In each of these cases the time of death of the deceased was known and the deceased was placed unclothed in the mummy position on a metal table in a constant temperature room.

Marshall & Hoares' "Most Acceptable Explanation"

1. During life a temperature gradient exists in the outer layers of the body.

2. At the time of death the temperature gradients in the deep layers of the body will be determined by this gradient.

3. Initially the gradient is so small and the body tissues such poor conductors of heat that the cooling in these regions will be negligible.

4. Any tendency to cool may be counteracted for a short time by continued heat production.

5. As skin temperature falls a steeper gradient will be distributed throughout the surface layers and the cooling rate in the deeper layers will increase.

6. As time passes the the gradients will become fully established and stabalize.

The Mathematical Expression Deriving From These Observations

1. Since the cooling of the body is being influenced initially by a phenomenon whose effect decays with time, Marshall and Hoare decided to approximate it with an exponential function:

$$Ce^{-pt}$$

2. Thus, the expression for the rate of cooling of a deceased body is:

$$\frac{d\theta}{dx} = -Z\theta + Ce^{-pt}$$

where t = time
θ = Body temperature – Ambient Temperature in º C
C, Z, p = constants for the deceased corpse under consideration

3. Using Derive to solve this first order differential equation solver with initial condition, θ = θ0

DSOLVE1(– Z·θ + Ce^{-pt}, –1, t, θ, 0, θ0)

which yields (ultimately):

$$\theta = \frac{Ce^{-pt}}{Z-p} + e^{-Zt}\left(\theta 0 - \frac{C}{Z-p}\right)$$

Marshall and Hoare's Example Case #84

#7: $C84 := \begin{bmatrix} 1 & 50.6 \\ 2 & 47.3 \\ 2.5 & 45.8 \\ 3 & 43.9 \\ 3.5 & 42.2 \\ 4 & 40.4 \\ 4.5 & 38.3 \\ 5.5 & 35 \\ 6 & 33.5 \\ 7.5 & 29 \\ 8.5 & 26.3 \\ 10.5 & 21.6 \end{bmatrix}$

Temperature differences here are given in degrees Farenheit
Ambient temperature is 43ºF

#8: $\mathrm{VECTOR}\left(\frac{\mathrm{LN}(C84_{i,2}) - \mathrm{LN}(C84_{i+1,2})}{C84_{i+1,1} - C84_{i,1}}, i, 1, 11\right)$

Note: the limiting value, Z, of z* appears to be 0.1

#9: [0.06744128, 0.064452408, 0.084739542, 0.078988198, 0.087180872, 0.10675977, 0.090101834, 0.087605245, 0.096166405, 0.09772689

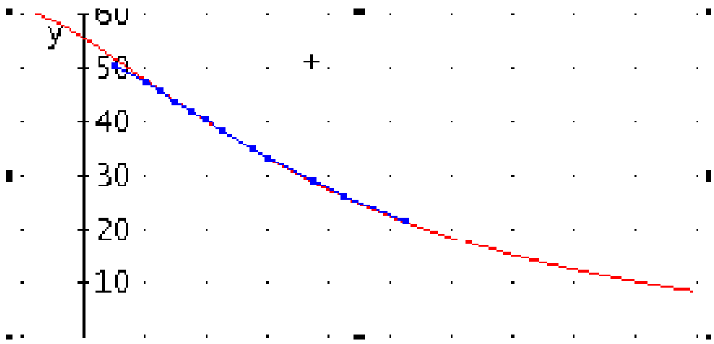
#10: $\theta = \frac{2.02 \cdot e^{-0.44 \cdot t}}{0.1 - 0.44} + e^{-t \cdot 0.1} \cdot \left(55.6 - \frac{2.02}{0.1 - 0.44}\right)$

#11: $\theta = 61.541176 \cdot e^{-0.1 \cdot t} - 5.9411764 \cdot e^{-0.44 \cdot t}$

Other values computed by Marshall and Hoare are calculated from the data:

C = 2.02, p = 0.44 B = θ0 – C/(Z – p) = 56.55

Graphical Agreement
Between Observed and Calculated Temperatures



Blue = observed excess temperature
Red = excess temperature calculated by the double exponetial model

Evaluating
Marshall and Hoare's Double Exponential Model

Advantages:

- Based on sound medical and physical analysis of the cooling procedure
- Very good accuracy when used in a laboratory setting

Disadvantages

- Very difficult to use on the scene of a death
- Requires several accurate temperature measurements
- Has gained virtually no acceptance among field practicioners
- Need to make assumptions about body size and mass
- Needs to be adjusted for body position and covering such as clothing and bed covers
- Assumes constant ambient temperature

Henssge Single Temperature Measurement Nomogram Based On Marshall And Hoare Equation

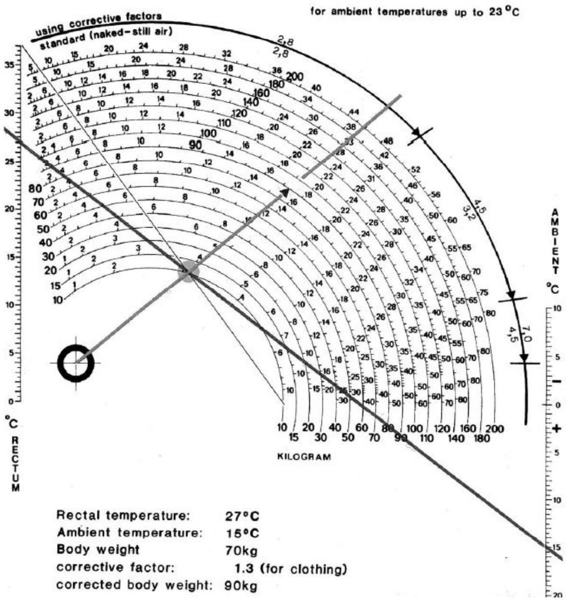


Table 3.16 Empirical corrective factors of the body weight for cooling conditions differing from the chosen standard[42]

Dry clothing/covering	Air	Corrective factor	Wet-through clothing/covering, wet body surface	In air/water
		0.35	Naked	Flowing
		0.50	Naked	Still
		0.70	Naked	Moving
		0.70	1–2 thin layers	Moving
Naked	Moving	0.75		
1-2 thin layers	Moving	0.90	2 or more thicker layers	Moving
Naked	Still	1.00		
1-2 thin layers	Still	1.10	2 thicker layers	Still
2-3 thin layers		1.20	More than 2 thicker layers	Still
1-2 thicker layers	Moving or	1.20		
3-4 thin layers	still	1.30		
More thin/	Without	1.40		
thicker layers	influence	–		
Thick bedspread		1.80		
		–		
+ clothing combined		2.40		
		–		
		3.00		

With corrective factor for clothing the estimate is 17.2 ±2.8 hours since death (95% confidence interval).

The Problem With Use of the Henssge Nomogram in the Field

1. Nomogram's are not widely used in today's technological society. They are a "mysterious" device. Since forensic practitioners are often called upon to testify in court, it would be difficult to explain its use within that context.

2. Their non-linear scale is unfamiliar to most non-scientifically sophisticated individuals.

3. Almost all school students who have had at least an introduction to algebra understand how to read the (x, y) coordinates of points on a graph, i.e. "given x, find y.

4. For the most part, field practitioners have not put Rainy's formulas into practice, and this adds another level of complexity.

5. While Henssge's basic idea: give practitioners a tool that can quickly be put to use in the field; is sound, the tool may not be appropriate for use within the context of today's technology.

Let’s return to the "double exponential model" for estimating the time since death:

$$\theta'(t) = -\beta t + \gamma * \exp(-\alpha t)$$

where: $\theta(t)$ is the difference between the observed body temperature and the ambient temperature (assumed to be constant)

β is the cooling rate according to Newton's Law of cooling

The Rainy Plateau is accounted for by the expression: $\gamma * \exp(-\alpha t)$

Solving Marshall and Hoare's differential equation with DERIVE,

$$\mathrm{DSOLVE1\_GEN}(-\beta\cdot\theta + \gamma\cdot\mathrm{EXP}(-\alpha\cdot t),\ -1,\ t,\ \theta,\ c)$$

$$\frac{e^{-t\cdot\alpha}\cdot\left(e^{t\cdot\alpha}\cdot\left(\theta\cdot e^{t\cdot\beta}\cdot(\alpha-\beta)-\gamma\right)+\gamma\cdot e^{t\cdot\beta}\right)}{\alpha-\beta} = -c$$

$$\mathrm{EXPAND}\left(\frac{e^{-t\cdot\alpha}\cdot\left(e^{t\cdot\alpha}\cdot\left(\theta\cdot e^{t\cdot\beta}\cdot(\alpha-\beta)-\gamma\right)+\gamma\cdot e^{t\cdot\beta}\right)}{\alpha-\beta} = -c,\ \mathrm{Rational},\ t\right)$$

$$\frac{\gamma\cdot e^{t\cdot(\beta-\alpha)}}{\alpha-\beta} + \theta\cdot e^{t\cdot\beta} - \frac{\gamma}{\alpha-\beta} = -c$$

$$\mathrm{SOLVE}\left(\frac{\gamma\cdot e^{t\cdot(\beta-\alpha)}}{\alpha-\beta} + \theta\cdot e^{t\cdot\beta} - \frac{\gamma}{\alpha-\beta} = -c,\ \theta\right)$$

$$\theta = \frac{e^{-t\cdot(\alpha+\beta)}\cdot\left(e^{t\cdot\alpha}\cdot\left(c\cdot(\alpha-\beta)-\gamma\right)+\gamma\cdot e^{t\cdot\beta}\right)}{\beta-\alpha}$$

$$\theta = \frac{\gamma\cdot e^{-t\cdot\alpha}}{\beta-\alpha} + \frac{e^{-t\cdot\beta}\cdot\left(c\cdot(\alpha-\beta)-\gamma\right)}{\beta-\alpha}$$

In 1988 Henssge (Forensic Science International v38, pp209) used a modified form of the solution to reduce the number of unknown constants to two. Note: *Q represents the remaining fraction of possible heat loss by the body at the time the rectal temperature is taken.:*

$$Q = \frac{T_r - T_a}{T_0 - T_a} = A \times e^{Bt} + (1-A)\times e^{\frac{A\times B}{A-1}t}$$

He and his team then did extensive research and statistical work to determine the values of A and B. The easiest to determine was the value of B since it was associated with the part of the cooling curve that is most directly related to the cooling coefficient in Newton's Law of Cooling. He found that this value was closely correlated to a persons body weight. Starting from the Rule of Surface given by Max Rubner, Henssge found the following relationship between body weight (in kg) and a statistically sound value for the constant (Not in general, but in terms of a particular case),

$$B(w) := -1.2815\cdot w^{-0.625} + 0.0284$$

The determination for the value of A required more insight. Typically, the bodies for study arrived in Henssge's lab 0.8 to 6 hours post mortem. The cooling plateau, the part of the cooling curve determined by A, lasts between 5 and 14 hours postmortem.

Henssge's insight was that there is a relationship between the rate of cooling during the latter portion of the process (Newton's Law of Cooling) and the rate during the earlier stages (plateau). He was able to produce the following statistically significant result for the cooling of bodies in ambient temperatures of less than 23 degrees Celsius.

$$Q = 1.25 \times e^{B(w)\times t} - 0.25e^{5\times B(w)t}$$

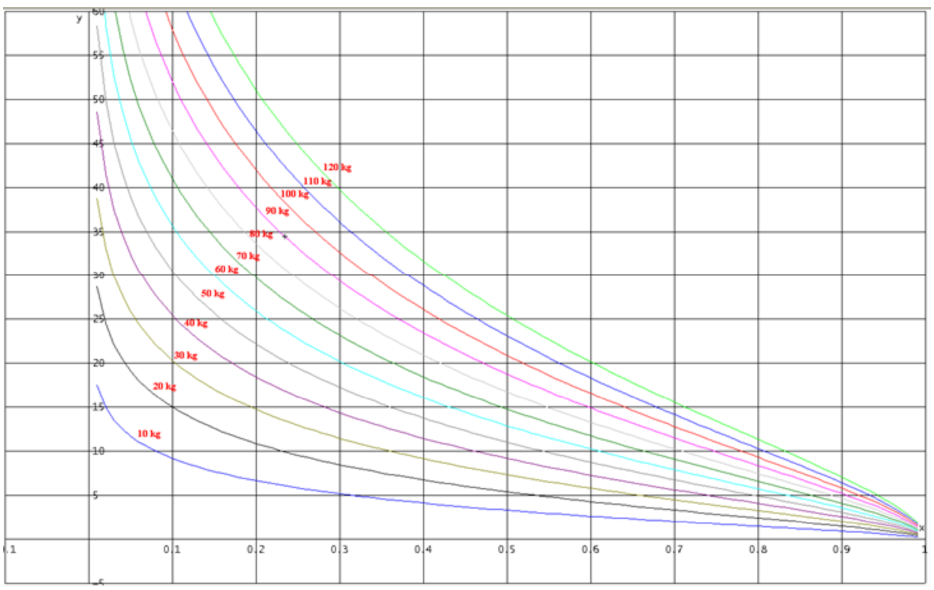
He also developed an analogous result for temperatures above 23 degrees. We leave this as a project for the "interested student".

In what follows we use DERIVE's numerical equation solver to draw the graphs of t, the time since death as a function of Q, the ratio of the retained body heat as reflected by the rectal temperature at the time the forensic investigator arrived on the scene of the death.

The following DERIVE definition gives the time estimates for Q = 0.99,…,0.1 in increments of 0.01 and body weights, w = 10,…,120 in increments of 10kg

$$C := \mathrm{VECTOR}\left(\mathrm{VECTOR}\left(\mathrm{NSOLUTIONS}\left(-\,0.25\cdot e^{5\cdot B(w)\cdot t} + 1.25\cdot e^{B(w)\cdot t} = \frac{100-j}{100},\ t,\ 0,\ \infty\right),\ j,\ 1,\ 99\right),\ w,\ 10,\ 120,\ 10\right)$$

$$\mathrm{VECTOR}\left(\mathrm{VECTOR}\left(\left[\frac{100-i}{100},\ C_{w,i,1}\right],\ i,\ 1,\ 99\right),\ w,\ 1,\ 12\right)$$



If the Forensic Investigators have a copy of this graph on their clipboard together with a copy of Henssge's adjustments for varying clothing and environmental conditions (see next slide), they can easily take one rectal temperature and then read the Marshall and Hoare estimate of the Time Since Death from the appropriate graph. The grid lines make the job easier.

Henssge's Results for Cases Other Than Ideal Conditions for Making Estimates

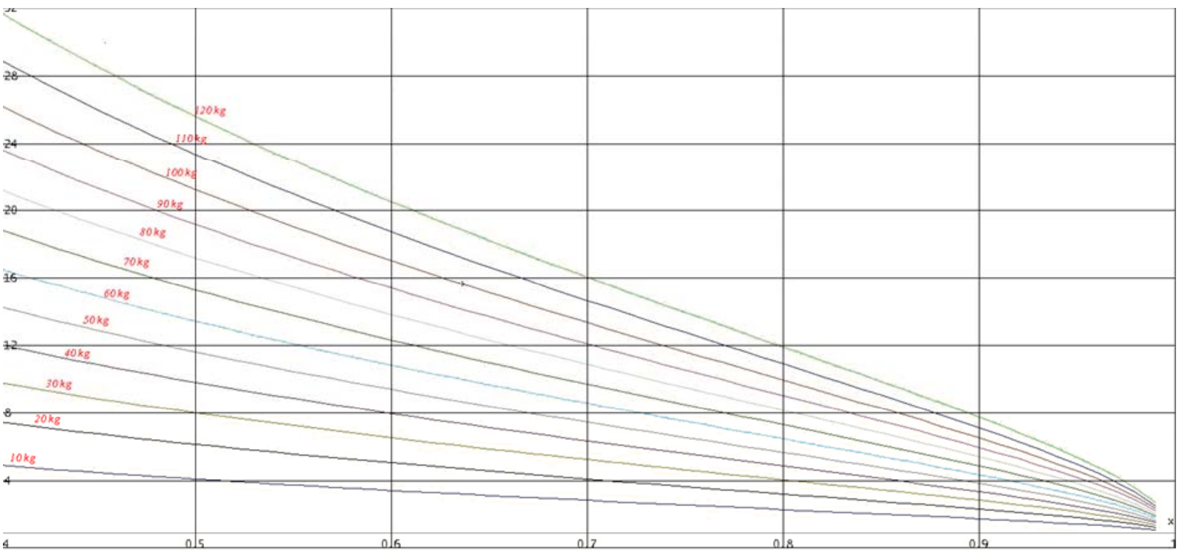
EMPIRIC CORRECTIVE FACTORS OF THE BODY WEIGHT*

Dry clothing/covering[a]	*In air*	*Corrective factor*[b]	*Wet-through clothing/covering wet body surface*	*In air/water*
		0.35	Naked	Flowing
		0.5	Naked	Still
		0.7	Naked	Moving
		0.7	1–2 thin layers	Moving
Naked	Moving	0.75		
1–2 thin layers	Moving	0.9	2 or more thicker	Moving
Naked	Still	1.0		
1–2 thin layers	Still	1.1	2 thicker layers	Still
2–3 thin layers		1.2	More than 2 thicker layers	Still
1–2 thicker layers	Moving or	1.2		
3–4 thin layers	Still	1.3		
More thin/thicker layers	Without influence	1.4 –		
Thick bedspread + clothing combined		1.8 – 2.4		

[a]Only the clothing or covering of the *lower trunk* is relevant!

[b]The listed Corrective Factors *for higher thermic isolation conditions* ($C = > 1.4$) are only valid for body weights between approx. 50 kg* and 80 kg*. In cases of lower body weights higher factors are necessary; in cases of higher body weights lower factors are necessary. More detailed investigation has to come.

In most coroner's cases the fraction of remaining body temperature loss, Q, is greater than .4. Thus we can produce a graph that is easier for the Forensic Investigator to read in the field.



This graph could easily be the second graph carried into the field by the Investigator.

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leinbach@gettysburg.edu