GeoGebra Workshop for the Initial Teacher Training in Primary Education

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ABSTRACT

This paper is part of a wider research we are conducting at the Faculty of Education (UAM) to discuss how to involve the dynamic geometry software GeoGebra in the development of geometric and pedagogic competencies in the degree Teacher in Primary Education.

We have designed a GeoGebra Workshop for the course "Mathematics and Methodogy II" (2nd year in the degree) to work the main geometric competencies that primary preservice teachers need to develop.

We will show examples of constructions made by the students in which we can analyze the development of skills such as reproduction of geometric figures, verification and generalization of properties, geometric problem solving, etc.

In addition, we try to know the impact of using dynamic geometry software in the student's attitudes and motivation.

Keywords

GeoGebra, Geometrical Competences, Pre-services Teachers, Dynamic Geometry.

Future teachers of elementary education in Spain must develop math skills that enable them to carry out its teaching properly. These skills include geometric competency, which usually offers many difficulties to our students.

The majority of preservice teachers reach the University with serious training shortcomings in the area of mathematics, which are even more accentuated in geometry contents. These contents are often relegated at the end of the agendas in other educational stages.

When students come to study the subject “Mathematics and Methodogy II” in 2nd course, that includes the contents of the block Geometry and Measurement, we find these problems of lack of understanding, difficulties with the specific vocabulary and deep insecurities.

We introduce this GeoGebra workshop to try our students to overcome the difficulties they find when they are solving geometric problems, and to facilitate their learning of concepts and relations among geometrical figures in a dynamic view. In addition, we believe that this way of working benefits them in their didactic competency.

Spanish educational legislation establishes in its curriculum of compulsory education the following guidelines:

Royal Decree 1513 / 2006 of 7 December lays down the minimum teachings of primary education; we find the area of mathematics content organized into four blocks: the third one corresponds to Geometry. We can read the overall objective of the study of this block of knowledge there:

On the study of the contents of the block 3, Geometry, students will learn about shapes and geometric structures. Geometry means to describe, analyze properties, classify and reasoning, and not only to define. Learning geometry requires thinking and doing, and should offer continuous opportunities to classify according to freely chosen criteria, to construct, draw, model and measure, developing the ability to visualize geometric relationships. All this is achieved, establishing constant relations with the rest of the blocks and other areas such as the world of art or science, but also by assigning a role to the manipulative part through the use of materials (geoboards and mecanos, grids of points, material to form Polyhedra, mirrors etc.) and personal activity performing bends, constructing, etc. to reach the concepts through actual models. The use of dynamic geometry software can contribute to that objective.

From this block there is a contribution to the development of mathematical competency mainly in the aspect of knowledge and management of basic mathematical elements (measures, symbols, geometric elements, etc.) in real or simulated situations of everyday life and the implementation of processes of reasoning that lead to the solution of problems or to obtain information. It is about that students

know how to develop skills and attitudes which allow them to reason mathematically and give a better response to situations of life in different levels of complexity.

Preservice teachers will therefore have to develop these skills to carry out its educational work. We think that GeoGebra can help our students because the dynamic geometry environment essentially produces a new kind of geometry. In this computer geometry, the figure is determined by the construction process and by how it then behaves under dragging (Forsythe, 2007).

On paper, a geometrical object such as a triangle is static. However, a triangle on a computer screen constructed using GeoGebra is rather different. It will not be a static triangle fixed in space, and how it behaves will depend on the method used to construct it. (Olive, 2000).

We go now to describe some activities developed in the GeoGebra workshop classified through these types:

- reproduction of constructions
- properties verification
- conjecture and research
- modeling of real-life situations

In this lecture we shall limit to expose some case studies on the first three types of activities that have been developed in a pilot workshop during the months of April and May of this year, with a group of students of the 2nd course of UAM Primary Education Faculty Diploma.

ACTIVITIES OF REPRODUCTION AND PROPERTIES VERIFICATION

In the practice 1, students must construct the triangle's perpendicular bisectors, altitudes, medians and angles bisectors. They must verify these lines intersect in a common point (circumcenter, orthocenter, centroid and incenter) and other properties.

In practice Nº 1 of the workshop we proposed the students the following activity:

(a) Construct a triangle ABC, draw the **perpendicular bisectors** of each side in red and verify that they intersect in a single point called Circumcenter. Write the name of the point also in red. Observe that if you draw a circle through A with center at that point, it also passes through the other two vertices: it's a circumference circumscribed to the triangle.

Add a *Check Box to Show/Hide Objects* that allows you to show/hide the bisectors, another for the circumcenter, and another one for the circumscribed circle.

(b) The ***altitude*** *corresponding to one side of a triangle is a segment perpendicular to the side through the opposite vertex. Every triangle has 3 altitudes.*

Draw the altitudes of triangle ABC in purple, and verify that they intersect in a single point called Orthocenter. Put the name of the point in the same color.

Add a *Check Box to Show/Hide Objects* that allows you to show/hide the altitudes and another one for the orthocenter.

(c) *The segment through a vertex and the midpoint of the opposite side is called the* ***median***. Draw the three medians of the triangle in green, and verify that they intersect in a single point called Centroid. Write its name in green. Measure the segments in which the centroid divides each median; what relationship do you observe with both segments of each median?

Add a *Check Box to Show/Hide Objects* that allows show/hide the medians and another one to the centroid.

(d) Draw the **angle bisectors** of the triangle in blue, and verify that they intersect in a single point called Incenter. Put its name in blue. If you draw through the incenter one perpendicular to each side, how much do the three formed segments measure?. Now draw circle with center the incenter and radius one of those segments (incircle).

Add a *Check Box to Show/Hide Objects* that allows you to view/hide the angle bisectors, another for the incenter, and another one for the incircle.

(e) Verify that, in any triangle, the circumcenter, orthocenter and centroid lay in a single straight line called **Euler's line**. Is the incenter in the Euler's line?

Add a *Check Box to Show/Hide Objects* that allows you to view/hide the Euler's line.

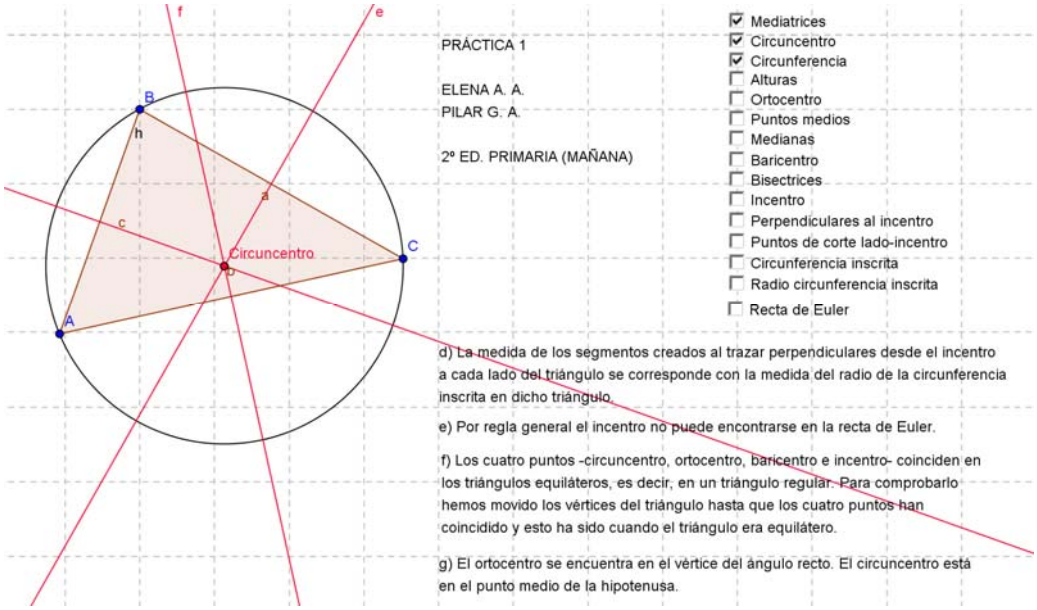
(f) Is there any triangle in which the four notable points (incenter, circumcenter, orthocenter and centroid) match, i.e. overlap? How have you seen it?

(g) In what position are there the orthocenter and the circumcenter of right triangles?

(h) Draw an isosceles triangle, knowing that the hypotenuse is 10 cm (track: consider properties that you have observed in the previous section).

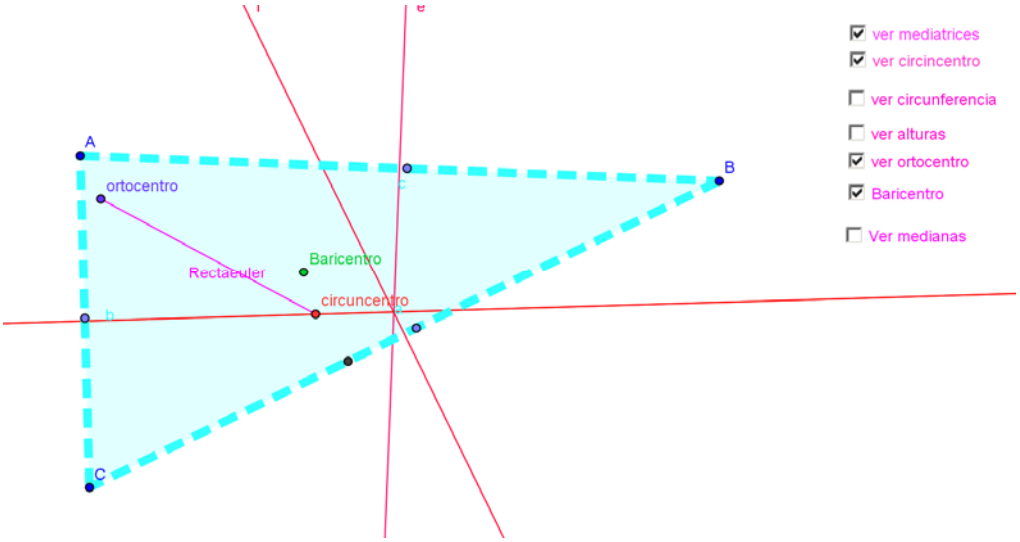
Activities a), b), c), d) and e) could be included in Reproduction and Verification of Properties category.

Reproduction and verification of properties activities are the simplest ones, in general. Most students perform correctly the reproduction part when they become familiar with GeoGebra tools.

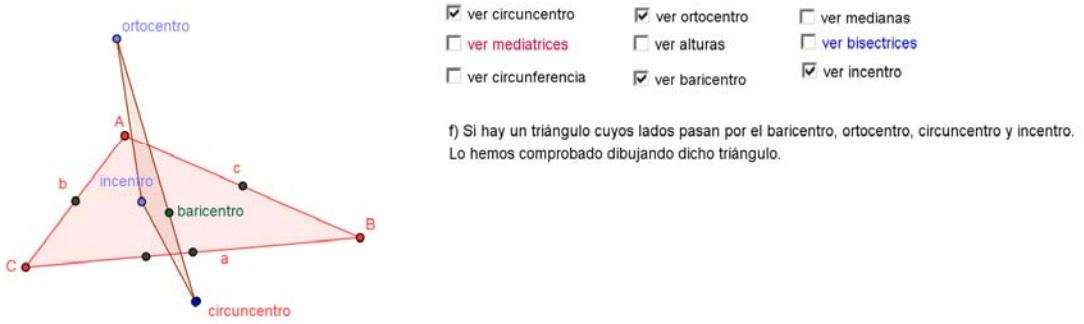


Some students commit the mistake of determining the midpoints of the sides of the triangle making a rough estimate, i.e. using the "New Point" tool instead of "Midpoint or Center", so they draw the medians and the centroid incorrectly.

We find the most frequent errors in the verification of properties, for example: using the "New Point" tool rather than "Intersect Two Objects", so that when students must verify that the lines always intersect in a single point, the construction is not maintained. This shows that the "dynamic" nature of GeoGebra is not exploited by students from the outset. Their long experience of static learning of geometrical concepts weighs too much and, in spite of the insistence of the teacher responsible for the workshop, they don't check their constructions at the end of the activities (they don't drag the vertices of the triangles to see if the properties are maintained). Implicitly, students work only with a triangle, not with "any triangle".



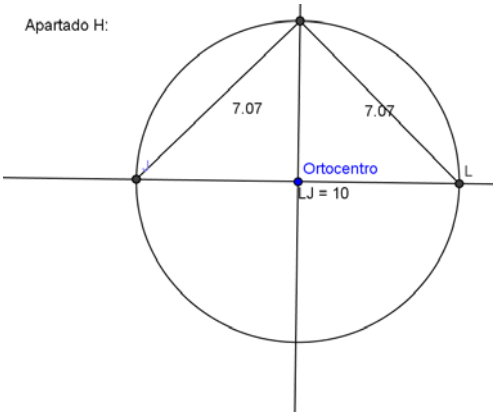
Another difficulty with some of the students is the geometric language comprehension. They don't make successful constructions because they interpret incorrectly the statements of the problems. This gap also affects activities where they must make justifications or formulate conjectures.



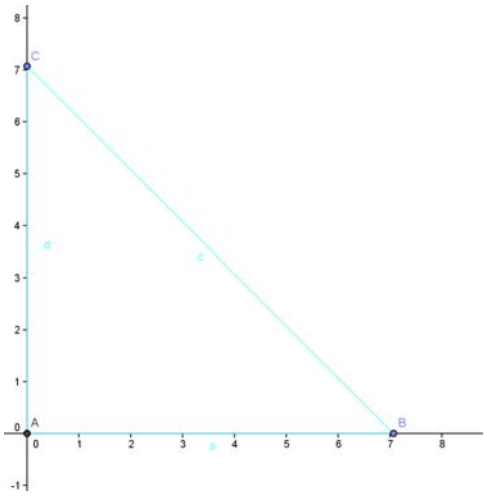
These students have incorrectly interpreted the question: *Is there any triangle in which the four notable points (incenter, circumcenter, orthocenter and centroid) match, i.e. overlap?* They have understood instead: I*s there any triangle passing through the four notable points?* In their activity there are many mistakes because they have used the "New Point" tool to determine all notable points, but these errors were not detected because they **haven't drag their triangle in any time. They have worked as if they had drawn static figures.**

ACTIVITIES OF CONJECTURE AND RESEARCH

An activity that has been very difficult for students of GeoGebra pilot workshop has been the paragraph h) in practice 1. It was making a construction of an isosceles right triangle whose hypotenuse is 10 cm. (It was intended to use the property which had been deducted in the g) paragraph): **the circumcenter in a right triangle is always at the midpoint of the hypotenuse.**



Although the majority of students answer correctly to the g) paragraph, they are not able to use this property to perform the construction ordered in h). Most of them force its initial triangle (which wasn't a right triangle) to have an "apparently 90º" angle and force their side's lengths the requested properties (some students conform to comply with one condition: the triangle is isosceles or the hypotenuse is 10 cm). Constructions lose all their properties when you drag one of the vertices of the triangle. (In this activity the workshop teacher had requested as a necessary condition for constructions to retain the properties of the triangle always).

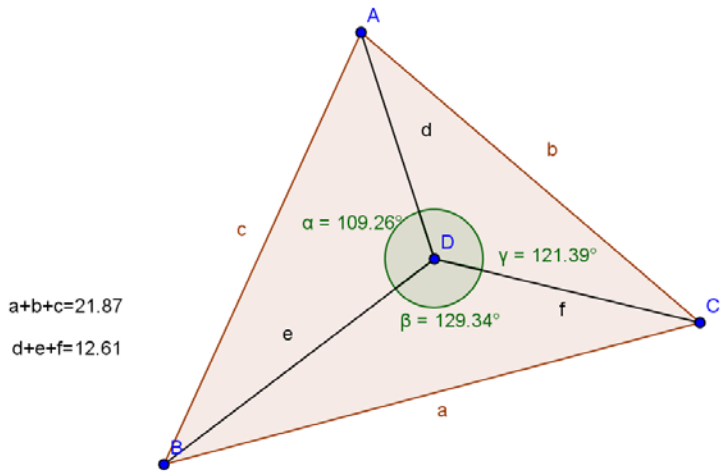


In this example, the student implicitly used the Pythagorean Theorem to determine that the other two sides should measure 7, 07 cm and builds the triangle "relying on the Cartesian axes". Her triangle does no longer keep the property of having a right angle by moving the vertices B or C. She hasn't used the property of the circumcenter.

More open research activities were also raised. Practice 2 called for:

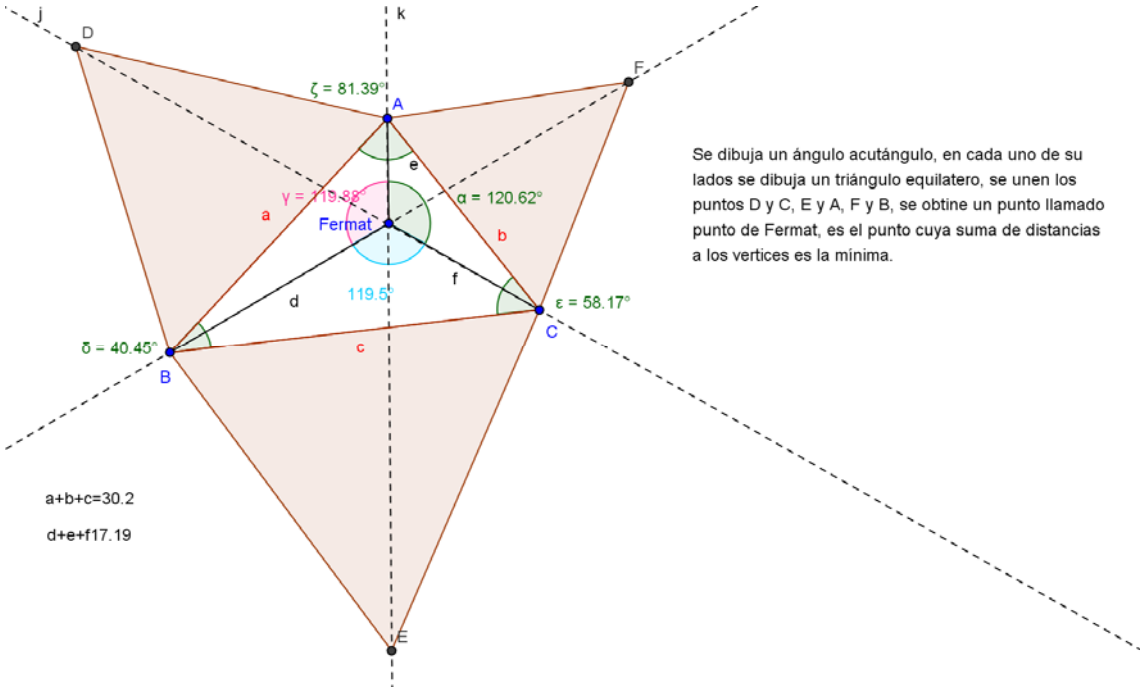
Practice 2:
Three cities are located in the three vertices of an acute-angled triangle. We want to build a road to join them so that the sum of the distances between the three cities is minimal. Search what conditions must be met for this.
Explain in detail the conclusions you have arrived at and the procedure you have followed.

We proposed to perform a construction to be used as a model for the problem. The first approach to the solution was to consider that the road could be drawn on the sides of the triangle formed by uniting three vertices of the city's position. The teacher's question was: Do you think that there may be a point inside the triangle from which the segments to the three vertices can form a shorter way? Students were invited to make a construction comparing both roads and arriving to formulate some property on the shortest path.



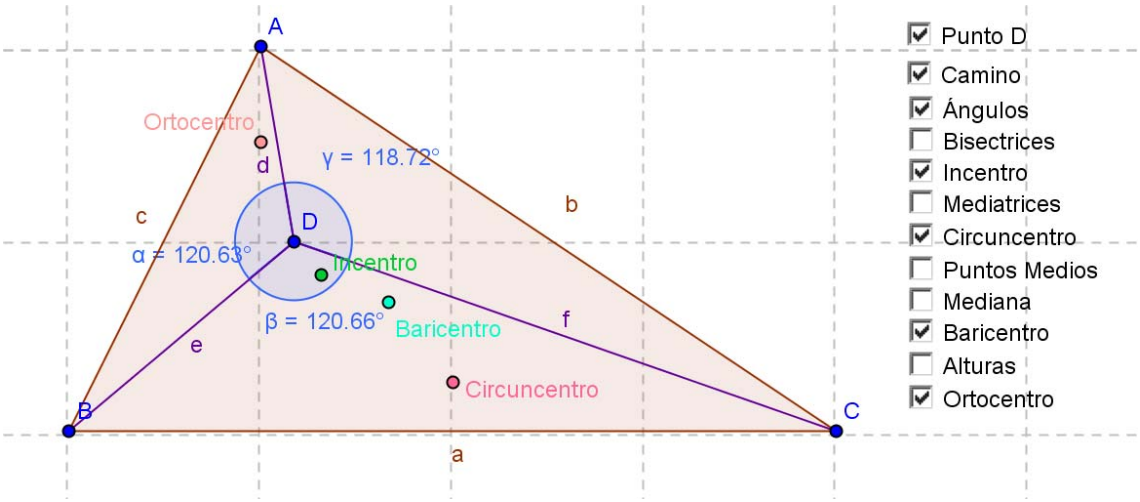
Most of the students found a relationship between the extent of α, β and γ angles: **the road was minimal when the three measured 120 °**

Some found that the so-called Fermat point was the solution to the problem and attempted the construction. Probably the students who came to this conclusion sought information to solve the problem on the Internet or other sources.



Many students tried to solve the problem building all or any of the 4 notable points of the formed triangle and seeing if any of them was the searched point. Some of them thought that the circumcenter, the centroid or the orthocenter could be the solution, but in these cases they didn't check more options. Almost all students that incorrectly

responded to the problem only tested on a fixed triangle; they didn't use the tool "drag" to test that the hypothesis was still serving in other cases.



It is remarkable the lack of rigor using the geometric language showed by students when they expressed the procedure followed and the conclusions they have arrived to.

SURVEY ON GEOGEBRA

At the end of the workshop, students who had assisted regularly were invited to fill a survey to know their opinion and suggestions. This information will help us to improve the design of the workshop we will implement the next school year with new 2nd course pupils of the Primary Education Diploma.

On a scale of 1 to 5, where 1 is "completely disagree" and 5 is "fully agree", please rate your level in accordance with the following statements:

Answer honestly, information will be used to design a workshop of GeoGebra the next course. Thank you.

2. GeoGebra is easy to use for me

3. I prefer to solve geometrical problems with pencil and paper than with GeoGebra

4. GeoGebra helps me to understand relationships between geometric objects

5. Working with GeoGebra is boring

6. With GeoGebra I can visually verify conjectures easily

7. I think that GeoGebra is not good to teach elementary geometry

8. GeoGebra adds something to the learning experience

9. I find it easier to block me with GeoGebra than with paper and pencil

10. GeoGebra helps me to explore, experiment and make conjectures

11. In the computer I prefer to work alone that in couple

12. With GeoGebra students are interested and understand what it is all about

13. Using GeoGebra I find it difficult to take the initiative to solve new problems

14. GeoGebra can help me to improve my geometric knowledge

15. I believe GeoGebra will help me to teach math to my students

16. Do you have any comments or suggestions regarding workshop of GeoGebra?

Analyzing collected responses, we see that preservice teachers believe that GeoGebra is easy to use (only a 14% are in disagreement). In preference to solve problems using GeoGebra or pencil and paper, almost half of the respondents are not defined by any option, which indicates that for them both methodologies are complementary.
It is very significant that 88% of the students respond with a 1 or a 2 to the statement "Working with GeoGebra is boring". It is also positively valued its use to encourage the geometric competences more difficult for them, such as research, formulation of conjectures and solving new problems.

Preservice teachers believe that GeoGebra can help them to improve their geometric knowledge and that it will be a useful tool in his future work as teachers of Primary Education (about an 80% responds both questions with 4 o 5).

Two suggestions appear many times to improve the GeoGebra workshop: to increase the number of sessions and its duration (this year each session had a duration of 1 h for reasons of availability of the computer room). Both improvements are planned for the next course, where the workshop will have at least 14 sessions of 1:30 h.

Other opinions and suggestions from students are:

"I found GeoGebra very interesting because it helps me to resolve doubts about concepts previously given"

"I liked a lot GeoGebra workshop and I have learned many things with it." "I think it's a very useful tool for learning and study geometry, as well as to use it in the classroom with my students in the future."

"It has been very short as I've found it very interesting; I would like to learn GeoGebra manage better and make more difficult things".

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