An example of learning based on competencies:
Use of Maxima in Linear Algebra for Engineers

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Adaptation to EAHE

- New attitude of teachers and students
- New methodology
- New material
- New model of assessment
- Long-life learning
- E, b and u-learning
Learning based on competencies

METACOMPETENCY

To be a good engineer integrated into society

Maths contribution

The ability to apply the mathematical techniques to solve engineering problems
Linear Algebra for engineers

- Our reference course: Open University in Spain (UNED) in 2009-2010 for the new grade in Computer Engineering
- A proposal of Linear Algebra course in e-learning model, with an integrated use of a CAS (for solving problems, calculations, etc.)
- Tutorials can be provided
Generic Competencies

- G1: Self Learning
- G2: Analysis and synthesis
- G3: Planning and organization
- G4: Communication and writing
- G5: Mathematical and technical writing
- G6: Use of technology
- G7: Information Management
- G8: Critical Thinking

For every math subject
Specific Competencies (in Linear Algebra)

- S1: Knowledge, understanding and use the basic concepts in Linear Algebra
- S2: Ability to apply knowledge, calculation and technology to solve mathematical problems for engineering
Assessment of Competencies

- For each competency several measurable indicators will be defined.
- Proper use of CAS is one of the indicators **G6: Use of technology**.
- Using the CAS also other competencies are developed.
Choosing the CAS

In our e-learning model we propose free software, which offers:

- Freedom to use it anywhere and for any purpose
- Freedom to study and adapt it to our needs
- Freedom to distribute it to students, which working at home
Essential requirements

- Easy of use
- Symbolic, numerical and graphical linear algebra features
- Accessibility and ease of installation
- Good maintenance
- Wide diffusion
The CAS wxmaxima

Maxima has interesting features in Linear Algebra
It is very easy to use and, being freely distributed, students can access it easily and have plenty of information
Material

- Textbook
- Study guide
- Worksheets
- Projects

- Maxima files
- On line self-assessment tests
- Worksheets
- Projects
Use of wxmaxima

- Software integrated into the course
- Students can use it in different ways:
  - To check calculations (early stage of learning)
  - To carry out routine mathematical procedures
  - To solve problems
Developing competencies by solving problems with Maxima

Before the use:
Training with Maxima

In the process of resolution

G1: Self Learning
G6: Use of technology

Other competencies
An example

- A surveillance device has access to images from security CCTV that focuses on the four sides of a building.

- The device is programmed in such a way that only shows one of the sides. After showing the same side for one minute it may “choose” to maintain the same image, with probability \( a \) (\( 0 \leq a \leq 1 \)) or may access one of the two adjacent sides of the building, with equal probability \((1- a)/2\). The security guard controlling the device introduces the value of \( a \), as a data.

- i) Which value of \( a \) should be introduced to display the same side constantly? (or to change always the controlled side?)
An example (2)

- ii) At 8:00 a.m the device displays the Nord side. The guard introduces the value $a = 1/2$. Find the probability of showing each of the sides at 9 a.m. Analyze the same problem with different values of parameter $a$. Pay special attention to the cases $a = 0$ and $a = 1$.

- iii) Study, for different values of the parameter $a$, the behavior of the device when $n$ minutes have passed, with $n$ very large.
Step 1: Modelling

- Analyzing the statement
- Identifying data and objectives
- Defining variables
- Choosing notation
- Looking for similar examples in references
- Proposing and validating the model
Output of step 1 in our example

From real world to Mathematical word

$V(n)$: Vector, after $n$ minutes of probabilities for staying in N-E-S-W

$V(n+1)=M \cdot V(n)$,

$$M = \begin{pmatrix}
    a & \frac{1-a}{2} & 0 & \frac{1-a}{2} \\
    \frac{1-a}{2} & a & \frac{1-a}{2} & 0 \\
    0 & \frac{1-a}{2} & a & \frac{1-a}{2} \\
    \frac{1-a}{2} & 0 & \frac{1-a}{2} & a
\end{pmatrix}$$

Mathematical problem: Computing $M^n$ for different values of $a$ and $n$
Step 2: Selecting concepts to be used

Matrix power

Diagonalization

• Eigenvalues and eigenvectors
• Similarity Matrix (P)
• Inverse Matrix
• Properties

G3: Planning and organization
G5: Mathematical writing

S1: Knowledge, understanding and use the principles of basic training in Linear Algebra

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Technology and its Integration into Mathematics Education. TIME 2010
Step 3: Resolution

- Introducing data at computer
- Making appropriate computations
- Exploring with different values
- Writing results

G5: Mathematical writing

S2: Ability to apply knowledge, skills and technology to solve problems

G8 Critical thinking

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Step 4: Interpretation of results

- Analyzing solutions
- Selecting outputs
- Translating results to real world

G8: Critical Thinking
G7: Information Management
S1
S2
G4: Communication and writing
Step 5: Application’s conditions and other alternatives

Verification of hypothesis

Defining procedures

Writing conclusions

G8: Critical Thinking

G5: Mathematical writing
G2: Analysis and synthesis

G4: Communication and writing
Step 6: Generalizations

G1: Self Learning

G2: Analysis and synthesis

G3: Planning and organization

G4: Communication and writing
G5: Mathematical writing

G8: Critical Thinking

S2: Ability to apply knowledge, skills and technology to solve problems
Our example (generalization)

- "Repeat" the experiment if you have the same device in a hexagonal building.
- Would it be possible to draw any conclusions for the position limit for a device located in a polygon with $h$ sides?
Homework

- Analyze, according the number of sides, the limit position in the case $a=0$
- There is difference between $h$ even or odd?
Conclusions

- The use of Maxima in solving problems may enhance several competences
- CAS in learning and assessment process:
  - i) Providing documentation for using
  - ii) Proposing the development of procedures for solving an algorithmic process
  - iii) CAS can be used in exams
THANK YOU

GRACIAS

COME TO MALAGA ON TIME 2010