A Computational Measure of Heterogeneity on Mathematical Skills

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Abstract

The educative fact is inherently multivariate, since there are lots of factors affecting each student and their performances. Due to this, both measuring of skills and assessing students are always complex processes. This is a well-known problem, and a number of solutions have been proposed by different specialists. But, in most of cases, it is clear that the different progress levels of students in the Mathematics classroom make also difficult the teaching work. We think that a measure of the heterogeneity of the different student groups could be interesting in order to avoid such difficulties, or to prepare some strategies to deal with this kind of problems.

The major aim of this work is to develop some new tools, complementary to the statistical ones that are commonly used for these purposes, to study situations related to education (mainly to the detection of levels on mathematical education) in which several variables are involved. These tools are thought to simplify and better understand these educational problems and, through this comprehension, to improve our teaching work.

Several authors in our research group have carried out some mathematical, theoretic tools, to deal with multidimensional phenomena, and applied them in business models. These tools are based on multidigraphs. In this work, we implement these tools by using symbolic computational software and apply them to study a specific situation related to the mathematical education.

1 Introduction

According to the Andalusian Education Law preamble (LEA [1]), 'the improvement experimented by the Andalusian Educative System in recent years cannot be denied'. However, the existence of aspects to be improved is also recognized within the same text. Detecting all these

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aspects is a complex task necessary to analyse and evaluate the Educative System. Besides, this evaluation has to be rigorous, exhaustive and objective.

The complexity of such analysis is obvious, and mainly due to the large amount of variables that involves. Although this number of variables makes the studies difficult, its importance is undoubted. A precise diagnosis could be the key to improve the education or to explain why certain plausible measures do not work in practise when taken.

Phenomena depending on several variables are called *multivariate*. Several papers prove that education is a multivariate phenomenon. Let us cite some examples: [4], [12], [13] and [14], all of them from official sources.

The above documents, and specially the *OECD Programme for International Student Assessment, PISA 2006*, consider several *analysis units* apart from multiple variables. For instance, to study the worldwide academic level, education in every country is considered, and conclusions are deduced from an interrelation of data. Even another subdivision can be considered, taking into account the so-called *components* of each analysis unit. For example, we can study the educative system in each Spanish region (more precisely, each autonomous community) in order to infer some conclusions about the national one.

Apart from these considerations, the educative phenomenon owns some other aspects that cannot be quantified [9]. But the description of these non-quantitative features exceed the goal of this paper. We only deal with some quantitative aspects of education in an alternative way to the statistical one, traditionally used.

This novel, topological approach was first developed in the Department of Economics, Quantitative Methods and Economic History, at Pablo de Olavide University, in Seville. Its seminal works are the following three:

In [10], Martín uses multidigraphs for a theoretical study of poverty and a more-in-depth analysis in some specific cases. In fact, she defines several topological indicators associated to these multidigraphs. The newest contribution of this topological technique is the simultaneous consideration of all the variables, taking always into account the multivariate feature of the phenomenon.

In [11], Mínguez continues Martín's work, applying topological indicators to analyze poverty again, from an economical point of view, and determining their stability.

In [15], Ballesteros, Hernández and Fedriani carry out a study about the development of mining heritage tourism in Andalusia, from the analysis of five mining Andalusian towns. Graph Theory is also involved in this evaluation.

Digraphs and topological models have also been used in Input-Output Analysis, a branch of Economics. Fedriani and Tenorio characterize in [6] the autonomous sets and the fundamental products of a given economy from the adjacency matrix of an associated digraph. Besides, they obtain some algorithms to find both autonomous sets and fundamental products and even implement some of them. In general, the use of multidigraphs can ease the implementation of algorithms.

Continuing with economical applications, in [8], Kaufmann and Gil-Aluja apply multidigraphs and neuronal networks to Economics and Business Management. In this work, multidigraphs and neuronal networks are related through the so-called neuronal graphs. They only apply this objects in an economical context, although further applications are also feasible, like diagnosis

in education, which is our main goal.
An important advantage of these topological techniques is that the multivariate feature is always present, because they allow us to work with all the variables at the same time and to make a comparison among all the analysis units within a case study (for instance, different countries, communities or educational centers). In this paper we apply the mentioned topological indicators to study the organization of groups in two secondary schools in Seville according to the results obtained in the diagnostic tests of mathematical capabilities carried out by the Consejería de Educación of the Junta de Andalucía during the 2007-2008 academic year [3].
Let us remark that, in the educative practice, the utility of these tools is twofold, comprising organization and teaching. From an organizational point of view, we can assure the design of more homogeneous or specific groups, since we can simultaneously study several features of the students, depending on the educational needs of the center. Didactically, they are useful tools to complement studies on Statistics along the Secondary period. Further didactic considerations are described beyond this paper.
Next, we are going to study the homogeneity of the 3rd course of ESO groups in two secondary schools in Seville, during the 2007-2008 academic year. Besides, we are going to arrange them in order of general skills in Mathematics by using three topological tools: the Intensity Topological Indicator, the Disparity Topological Indicator and the Competence Levels. Our data source is the Consejería de Educación of the Junta de Andalucía [3].

2 Intensity Topological Indicator

In this section, the Intensity Topological Indicator will be defined and later applied to analyse the diagnostic tests of mathematical capabilities for students on 3rd course of ESO in two secondary schools in Seville during the 2007-2008 academic year. The data base has been provide by the Consejería de Educación of the Junta de Andalucía [3], although we are not authorized to publish the names of these centers. One of them (from now on, CENTER 1) lies in the center of the city and the other one belongs to a rural area of the province (our CENTER 2).
The diagnostic tests are applied to all students on 3rd course of ESO in Andalusia, and try to measure the mathematical skills that students have got after the first two years in a secondary comprehensive school.
Terminology and notation on Graph Theory can be found in [5, 7], although all the concepts used will be defined here, in order to ease the reading of this paper.
Let us define the *universe* U of our study as the set of students on 3rd course of ESO in CENTER 1 or CENTER 2 during the 2007-2008 academic year.
Let us split U into two *analysis units*: C_1 and C_2. C_1 is formed by the students of U in CENTER 1, while C_2 refers to the students of U in CENTER 2. Obviously, $C_1 \cup C_2 = U$ and $C_1 \cap C_2 = \emptyset$, so $\{C_1, C_2\}$ is a partition of U.
Besides, for every analysis unit, let us define a partition set whose elements will be called *components*. So $3A$, $3B$, $3C$ and $3D$ are the components of C_1 defined as the sets of students in CENTER 1 belonging to groups A, B, C and D, respectively. Analogously, let $3A'$, $3B'$, $3C'$

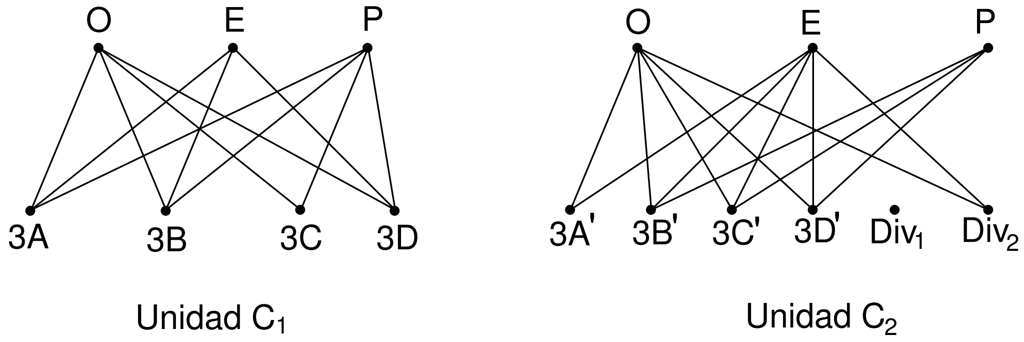

Figure 1: Bipartite graphs associated to C_1 and C_2.

and $3D'$ be the components of C_2 with the students in the groups of CENTER 2 (again, 3rd of ESO A, B, C and D, respectively. As there are two more groups of students in 3rd course of ESO in CENTER 2, corresponding to a special level called *Diversificación Curricular*, we obtain another two components denoted by Div_1 and Div_2. Students in these two groups are pupils with detected learning difficulties.

Hence, we have that C_1 components are $\{3A, 3B, 3C, 3D\}$ and C_2 ones are $\{3A', 3B', 3C', 3D', Div_1, Div_2\}$.

Now, let us consider three dimensions (or variable names) included in our data source. These dimensions are defined as: *organizing, understanding, and interpreting information*, denoted by O; *mathematical expression*, denoted by E; and *planning and solving problems*, denoted by P. These variables take numerical values from 1 to 6.

In a preliminary step, for any variable, a *threshold* is defined. Its use will be explained later on. For every variable, we have chosen (as our threshold) the arithmetic mean of the means of all values taken by this variable in each analysis unit. Their values has been computed from our data source [3], obtaining the following: *threshold for* $O = 4$; *threshold for* $E = 3$; *threshold for* $P = 4$.

In general, we consider that a unit is *competent* with respect to a given variable if the value of the variable for this unit is *greater than or equal to* the threshold fixed for this variable. Of course, some variables may take values greater than or equal to their threshold (and some others not) in a same unit, and this may happen in each unit. We need some graph tools to establish an order according to this situation.

Let us define a bipartite graph for every analysis unit (see [11]). A bipartite graph has its vertex set divided into two classes, and any edge has its extremes into two different classes. In our case, the first vertex class is $\{3A, 3B, 3C, 3D\}$ for C_1 and $\{3A', 3B', 3C', 3D', Div_1, Div_2\}$ for C_2. In both cases, the second class is $\{O, E, P\}$.

The edges are defined as follows. A vertex in the first class is linked to another one in the second class if the value of the represented variable in the represented component is greater than or equal to its threshold. The resultant graphs are shown in Figure 1.

Note that the above construction is based on Table 1, extracted from our data source [3].

	3A	3B	3C	3D
O	5	4	5	5
E	3	3	2	3
P	4	5	4	4

	3A'	3B'	3C'	3D'	Div_1	Div_2
O	4	4	4	5	3	4
E	3	3	3	4	2	3
P	3	4	4	5	2	3

Table 1: Values of O, E and P, for C_1 and C_2.

The Intensity Topological Indicator (ITI) is defined as follows (see [10, 11] for further details).

Definition 1. *Let G be the bipartite graph associated to the analysis unit C, with $V(G) = V_1 \cup V_2$, where V_1 is the vertex class whose elements are the analysis unit components and V_2 is the vertex class whose elements are the considered variables. Then, the Intensity Topological Indicator is defined as* $ITI(C) = \dfrac{\#(E(G))}{\#(V_1)}$.

The classical notation from Graph Theory is also used here (see [5, 7]), where $V(G)$ is the set of vertices of a graph G, and $E(G)$ is the set of its edges. Besides, $\#(A)$ is the cardinal of set A, that is, how many elements A has.
From Definition 1, we have that $0 \leq ITI(C) \leq \#(V_2)$; the more $ITI(C)$ approaches 0, the less intensive is the considered property in C.
We have obtain that, for C_1, $\#(A(G)) = 11$ and $\#(V_1) = 4$ (see Figure 1), so $ITI(C_1) = \dfrac{11}{4} = 2.75$ $(0 \leq ITI(C_1) \leq 3)$.
For C_2, $\#(A(G)) = 13$ and $\#(V_1) = 6$ (Figure 1), so $ITI(C_1) = \dfrac{13}{6} = 2.17$ $(0 \leq ITI(C_2) \leq 3)$.
It is advisable the use of some other indicators before coming to a definitive conclusion but, from ITI, we can infer that **both analysis units are slightly under the average level of mathematical competence**, because the average value is $ITI = 3$, and the order according to maths skills is $C_2 \prec C_1$, since $ITI(C_2) = 2.17 < 2.75 = ITI(C_1)$.
We can complete this preliminary analysis. In [10, 11], the partial indicators with respect to every variable are also defined:

Definition 2. *Based on the indicated notation in Definition 1, the partial Intensity Topological Indicator with respect to variable T (ITI_T) is defined as $ITI_T(C) = \dfrac{\delta(T)}{\#(V_1)}$, where $\delta(T)$ denotes the degree (or valency) of vertex T, that is, the number of edges which are incident with T (see [10, 11]).*

From Definition 2, we immediately have that $\sum_{i=1}^{k} ITI_{T_k}(C) = ITI(C)$, where $T_1, ..., T_k$ are the considered variables. In addition, we have that $0 \leq ITI_T(C) \leq 1$.
In our case, the values obtained for the partial indicators applied to C_1 and C_2 are the ones in Table 2.
Note that only ITI_E is less than 1 in C_1, whilst all the partial indicators are under 1 in C_2. Besides, the order with respect to variable E is $C_1 \prec_E C_2$, whereas the order with respect to the

Unit	ITI_O	ITI_E	ITI_P	ITI
C_1	$\frac{4}{4} = 1.000$	$\frac{3}{4} = 0.750$	$\frac{4}{4} = 1.000$	2.750
C_2	$\frac{5}{6} = 0.835$	$\frac{5}{6} = 0.835$	$\frac{3}{6} = 0.500$	2.170

Table 2: Partial indicators for C_1 and C_2.

Unit	ITI_O	ITI_E	ITI_P	ITI
C_1	100 %	75 %	100%	91.67 %
C_2	83.5 %	83.5%	50%	72.23 %

Table 3: Partial indicators for C_1 and C_2, in relative terms.

other two variables is $C_2 \prec_{O,P} C_1$. The indicator has already been globally studied. Indicators can also be expressed in relative (percentage) terms as Table 3 shows.

3 Disparity Topological Indicator

We have arranged our analysis units in order of mathematical skills, by using the ITI. Besides, we have determined the global competence level of each one with respect to the thresholds of the considered variables.
But it would be also desirable to establish different levels of competence and classify every component into these levels, to get an idea of the homogeneity of the components. We can do this by using the *Disparity Topological Indicator* (DTI). In our study, we will check the homogeneity of the groups in the studied centers and the differences between groups in mathematical competence.
So, let us consider each unit separately to establish a homogeneous competence level in another example also related to education.

3.1 Associated Multidigraph Construction

We are going to associate a multidigraph (directed graph with multiple edges) to each unit, separately. In our case, we denote these multidigraphs by $G(C_1)$ and $G(C_2)$, respectively, and they are defined as follows: each set of vertices (usually called nodes, when dealing with digraphs) will be the set of components of every unit, that is, $V(G(C_1)) = \{3A, 3B, 3C, 3D\}$ and $V(G(C_2)) = \{3A', 3B', 3C', 3D', Div_1, Div_2\}$. The considered variables will be the same introduced in the above section, that is, O, E and P. Finally, the edges (or arcs, in the case of multidigraphs) of $G(C_1)$ and $G(C_2)$ are defined in the following way: vertices i and j are connected if the value of any variable at i is greater than or equal to the one at j. In a practical sense, this means that group i is more competent than group j in the aspect represented by the considered variable.

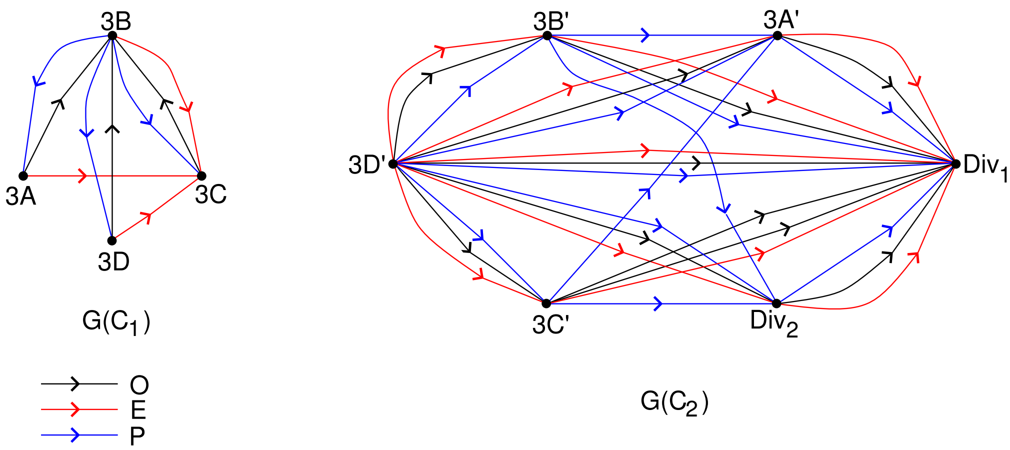

Figure 2: Multidigraphs associated to C_1 and C_2.

Sometimes, it could be advisable (not necessary, though) to distinguish among the edges according to the variable they come from.
Figure 2 shows multidigraphs $G(C_1)$ and $G(C_2)$.

3.2 Competence Levels

As insinuated before, components in every unit are going to be classified into levels such that components in a given level have a 'similar' mathematical competence and form a homogeneous block. Besides, all these levels will be ordered from higher to less mathematical competence.
Obviously, the more homogeneous an analysis unit is the less the number of levels it has. The ideal case of homogeneity takes place when a unit has only one level where all its components are in. In a practical sense, we may use this information to determine whether students has been homogeneously distributed in groups according to their mathematical skills.
A useful tool to deal with the complex multidigraphs shown in Figure 2, is their *adjacency matrix*, that is, a square matrix whose order is the number of vertices of the multidigraph and whose entry m_{ij} is the number of directed edges from vertex i to vertex j (see [5, 7] for further details). The adjacency matrices associated to multidigraphs $G(C_1)$ and $G(C_2)$ are, respectively:

$$M(C_1) = \begin{pmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 2 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \end{pmatrix} \qquad M(C_2) = \begin{pmatrix} 0 & 0 & 0 & 0 & 3 & 0 \\ 1 & 0 & 0 & 0 & 3 & 1 \\ 1 & 0 & 0 & 0 & 3 & 1 \\ 3 & 3 & 3 & 0 & 3 & 3 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 3 & 0 \end{pmatrix}$$

In [8], Kaufmann and Gil-Aluja define the *transitive closure* of a graph as another graph whose adjacency matrix is the boolean addition of the boolean powers of the original graph adjacency matrix, from 1 to the number of vertices of this graph. The boolean addition of two matrices is defined as another matrix whose entries are all 0 or 1. An entry is 0 when the correspondent element in the ordinary addition is 0, and 1 otherwise. The boolean product of two matrices is defined in a similar way, and a boolean power (of natural exponent) is defined as the boolean product of a matrix by itself as many times as indicated by the exponent. Matrices of the transitive closures of $G(C_1)$ and $G(C_2)$ are:

$$\hat{M}(C_1) = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{pmatrix} \qquad \hat{M}(C_2) = \begin{pmatrix} 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}$$

A graph is said to be *strongly connected* if any two distinct vertices are connected by a path. A strongly connected subgraph is *maximal* if it is not strictly contained in another strongly connected subgraph (see [5, 7, 8] for further details). Obviously, a strongly connected graphs has just one maximal strongly connected subgraph, the whole graph.
In our practical case, all components in a same maximal strongly connected subgraph are considered with a similar level of competence, because every component is connected to the rest. So, to determine the competence levels is related to find the maximal strongly connected subgraphs, to some extent.
Every maximal strongly connected subgraph can be obtained from the transitive enclosure of a graph. Indeed, for any vertex X_i, the maximal strongly connected subgraph where X_i lies is:

$$C(X_i) = (\hat{\Gamma}(X_i) \cap \hat{\Gamma}^{-1}(X_i)) \cup \{X_i\} \tag{1}$$

where $\hat{\Gamma}(X)$ is the set of vertices X_j such that there exists a path from X to X_j (in the adjacency matrix of the transitive enclosure, entry $\hat{m}_{ij} = 1$), and $\hat{\Gamma}^{-1}(X)$ is the set of vertices X_j such that there exists a path from X_j to X (in the adjacency matrix of the transitive enclosure, entry $\hat{m}_{ji} = 1$).
Therefore, a graph is strongly connected if and only if every entry of the adjacency matrix of its transitive enclosure equals 1.
In our case, $\hat{M}(C_1)$ is a matrix whose entries (all of them) equal 1, so $G(C_1)$ is strongly connected. Thus, unit C_1 has components with a similar level of mathematical competence. Hence, groups of the 3rd course of ESO can be said to be quite homogeneous with respect to mathematical skills and, for this reason, we cannot distinguish better or worse groups. Notice that, according to many specialists, grouping student in groups with similar levels is considered better than getting groups of 'good' and 'bad' students. This last statement may be questioned but, in any case, this controversy is beyond the goal of this paper. Our aim is only to measure the level of homogeneity among different groups, and this information can be useful to whom it may concern the management of the involved center.

However, the obtained maximal strongly connected subgraphs for $G(C_2)$, by applying 1, are the induced by the vertices $\{3A'\}$, $\{3B'\}$, $\{3C'\}$, $\{3D'\}$, $\{Div_1\}$ and $\{Div_2\}$; that is, there are not maximal strongly connected subgraphs with more than one vertex.
As commented before, components lying in a same maximal strongly connected subgraph are 'similarly' competent. But components in two distinct maximal strongly connected subgraphs can also have the same level of competence. Thus, it would be desirable to define a set of *levels* or *layers* that fulfill the following conditions:

1. Layers must be disjoint, that is, each vertex must be in a unique layer.

2. The union of all layers must be the whole graph.

3. Layers must be *totally ordered*. From a practical point of view, this means that the level of competence for two distinct layers must be always comparable.

The following algorithm can be applied to split a given graph, G, into layers:
Firstly, an associated graph G' is built by contracting to a point (a vertex) every maximal strongly connected subgraph (see further details about the contraction of graphs in [5, 7]). The obtained graph, G', has no cycle.
Secondly, the following algorithm (by Kaufmann and Gil-Aluja) [8] is applied on G': Let layer N'_0 be the set of vertices of G' such that no edge ends on them. Layer N'_1 is defined as the set of vertices of $G' - N'_0$ such that no edge ends on them. Layer N'_2 is the set of vertices of $G' - (N'_0 \cup N'_1)$ such that no edge ends on them, and so on. This is a finite process because at least a vertex is removed from G' in every step and G' is assumed to be finite.
Finally, let N_k be the set of vertices of G lying in a maximal strongly connected subgraph contracted to a point of N'_k in G'.
Graph $G(C_1)$ is strongly connected, so only layer N_0 is defined when the algorithm is applied to it, and N_0 is the set of vertices of $G(C_1)$. Implications of this result has been commented above.
Graph $G(C_2)$ has no maximal strongly connected subgraph with more than one vertex, so Haufmann and Gil-Aluja's algorithm is applied directly, without any previous contraction. Figure 2 shows $3D'$ as the only vertex not being an edge-end, so $N_0 = \{3D'\}$. When deleting this vertex, there is no edge ending on $3B'$ or $3C'$, so $N_1 = \{3B', 3C'\}$. Continuing this process, we have that $N_2 = \{3A', Div_2\}$ and $N_3 = \{Div_1\}$.
Recall that the origin vertex of an edge has a greater level in competence than its end, with respect to the considered variable and according to our construction. Thus, we can arrange the following order among the obtained layers:
$\{3D'\} \prec \{3B', 3C'\} \prec \{3A', Div_2\} \prec \{Div_1\}$
and our practical interpretation could be the following:
Four distinct levels in mathematical competence can be observed in C_2. Distribution of 3rd ESO students in groups is not homogeneous in this center.
The group associated to $3D'$, that is 3rd ESO D, presents the highest mathematical competence. Thus, students of 3rd course of ESO with the best skills in Mathematics can be found there.

The group associated to Div_1 (*Diversificación Curricular 1*) presents the worse difficulties in Mathematics (or the lowest competence level), which could match with the special features of this educative program.
The groups associated to $3A'$ and Div_2, respectively, lie in the same layer, so their students' mathematical skills are similar. However, students under *Diversificación Curricular* program are suppose to have special difficulties or a lower competence; so, using this information, the correct choice of this group for *Diversificación Curricular* should be questioned.
A priori, only two layers should have been expected for C_2: $\{3A', 3B', 3C', 3D'\}$ and $\{Div_1, Div_2\}$, in this order. The obtained result lead us to reconsider the grouping of students in 3rd course of ESO for this center and, consequently, the selection of students for for *Diversificación Curricular* project.

3.3 Disparity Topological Indicator

This indicator tries to measure the level of homogeneity of a given analysis unit and it is based on the previous computations of layers.

Definition 3. *Let G be the multidigraph associated to a given analysis unit, as it is shown in the former section. The Disparity Topological Indicator (DTI) is defined as $DTI(G) = \dfrac{k}{\#V(G)}$, where k is the number of layers obtained from the above algorithm.*

Note that $DTI(G) \in (0, 1]$, and the more homogeneity in the unit the closer to 0 the index. This indicator can also be expressed in percentage terms.
In our study, $G(C_1)$ has 1 layer and 4 vertices, so $DTI(G(C_1)) = \dfrac{1}{4} = 0.25 = 25\%$, whereas $G(C_2)$ has 4 layers and 6 vertices, giving $DTI(G(C_2)) = \dfrac{4}{6} \approx 0.67 = 67\%$. Realise that $G(C_1)$ is closer to 0 than $G(C_2)$, so a greater homogeneity is expected in C_1.
Eventually, let us remark that ITI and DTI indicators, the construction of the bipartite graph and the multidigraph associated to a given analysis unit, and the algorithm to split into layers has been successfully implemented using Maxima, a freely distributed symbolic calculus program available at any Guadalinex distribution. This can make easier the use of these topological tools in educative centers, especially in Andalusia.

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