

## ***Time 2010 MALAGA***

On the visualization of the calculus concept

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## **Novi Sad, Serbia**





## On the visualization of the calculus concept

Calculus is built on combining

**graphic, symbolic and analytic** representations of functions

Using computer software and graphical calculators

**graphic, symbolic, and numeric**

The influence of computer on the transition to  
advance mathematical thinking



## On the visualization of the calculus concept

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All 24 students were familiar with programme package GeoGebra

- The students are third grade, talented,
- used to make different visualization for the mathematical notion,
- drawing the graphs of functions,
- limits of sequences,
- limits of functions,
- continuity of functions



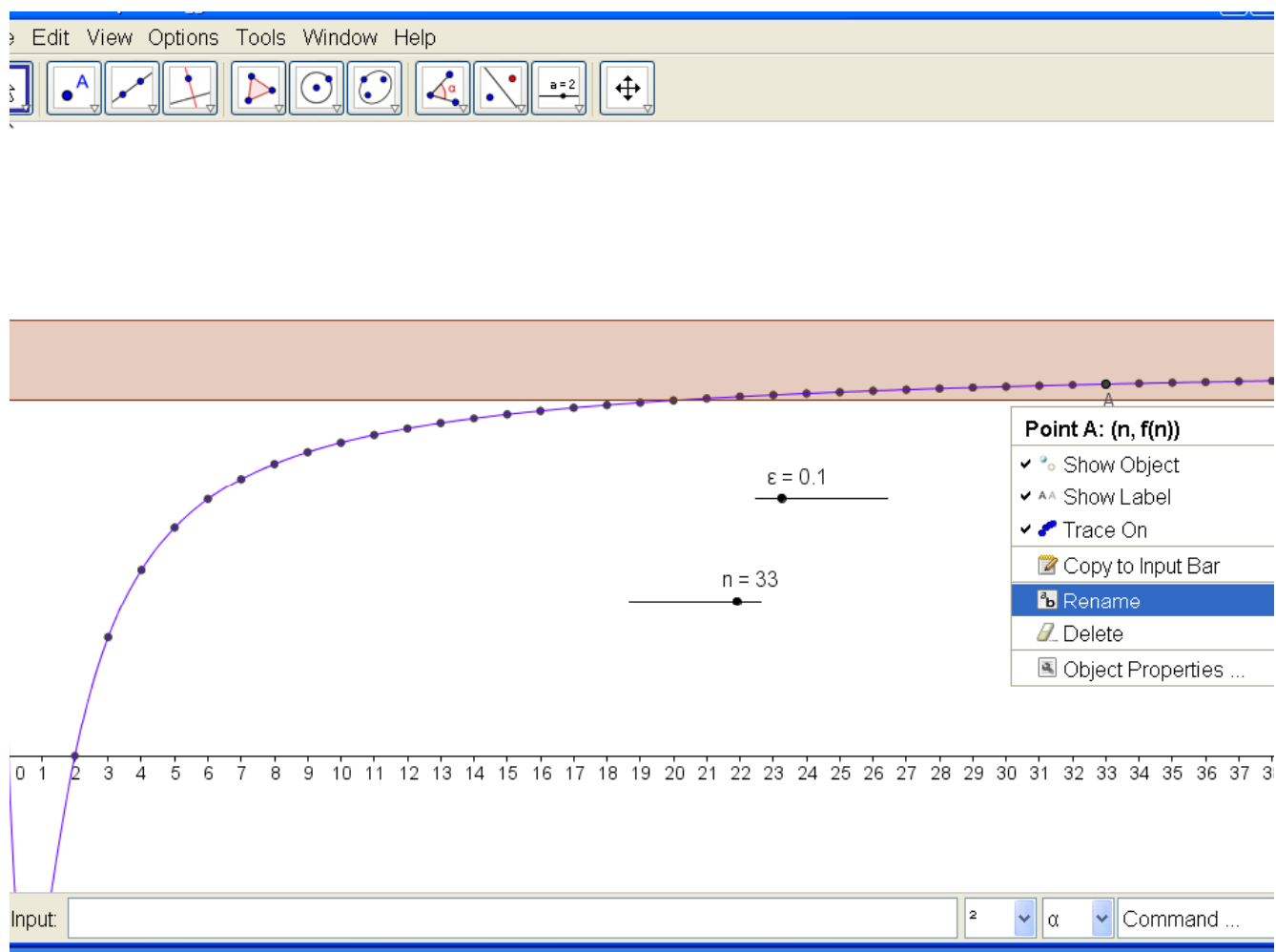
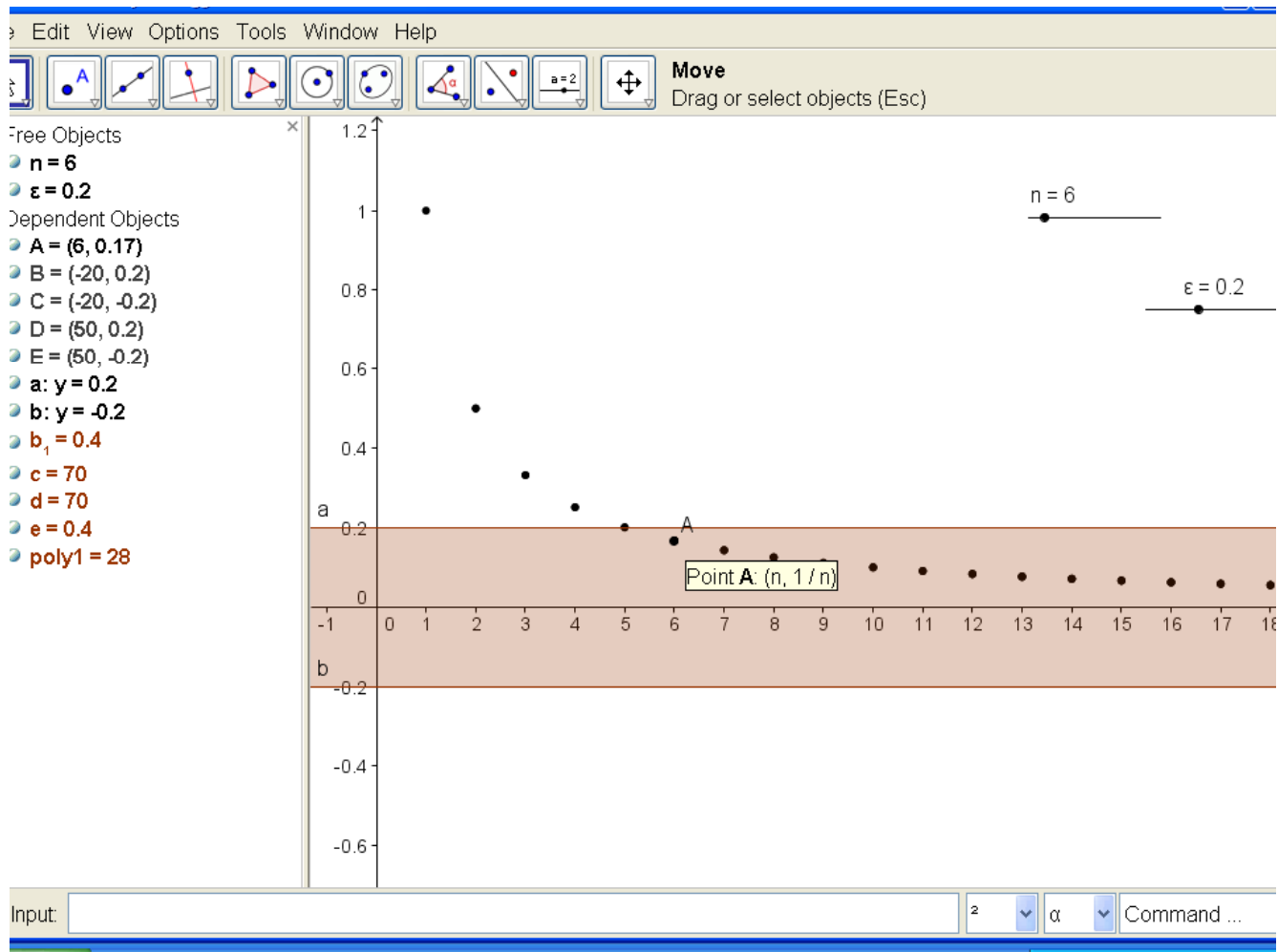
## On the introduction of the limits of sequence

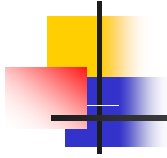
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We present the students work on determining

$$a) \quad \lim_{n \rightarrow \infty} \frac{1}{n}$$
$$b) \quad \lim_{n \rightarrow \infty} \frac{n^2 - 2n}{n^2 + 1} = 1$$

[Mhajlo-niz.ggb](http://Mhajlo-niz.ggb)  
[Arpad-niz.ggb](http://Arpad-niz.ggb)





- Student Mihajlo used the sliders  $n$ , for changing  $n$  and point  $A(n, 1/n)$  with the command *Trace On*.
- As the visualization of limit process he used  $\varepsilon$  –lane

Arpad first draw the corresponding function

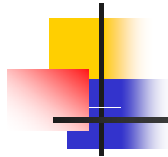


## On the visualization of the calculus concept

Introduction of the definition of derivative

$$\lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

The students got the following task:



## The questionnaire

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Determine, with and without computer the following limits,

1)  $\lim_{h \rightarrow 0} \frac{(x+h)^2 - x^2}{h};$

2)  $\lim_{h \rightarrow 0} \frac{\sqrt{x+h} - \sqrt{x}}{h};$

3)  $\lim_{h \rightarrow 0} \frac{(x+h)^a - x^a}{h}, \quad a \in R;$

4)  $\lim_{h \rightarrow 0} \frac{e^{x+h} - e^x}{h};$

5)  $\lim_{h \rightarrow 0} \frac{a^{x+h} - a^x}{h}, \quad a \in R;$

6)  $\lim_{h \rightarrow 0} \frac{\sin(x+h) - \sin x}{h}$

7)  $\lim_{h \rightarrow 0} \frac{\cos(x+h) - \cos x}{h};$

8)  $\lim_{h \rightarrow 0} \frac{\ln(x+h) - \ln x}{h}$

$x = 1, \quad x = a,$

or for arbitrary  $x$



## On the visualization of the calculus concept

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They were surprised: function of two variables?

How to apply limit process?

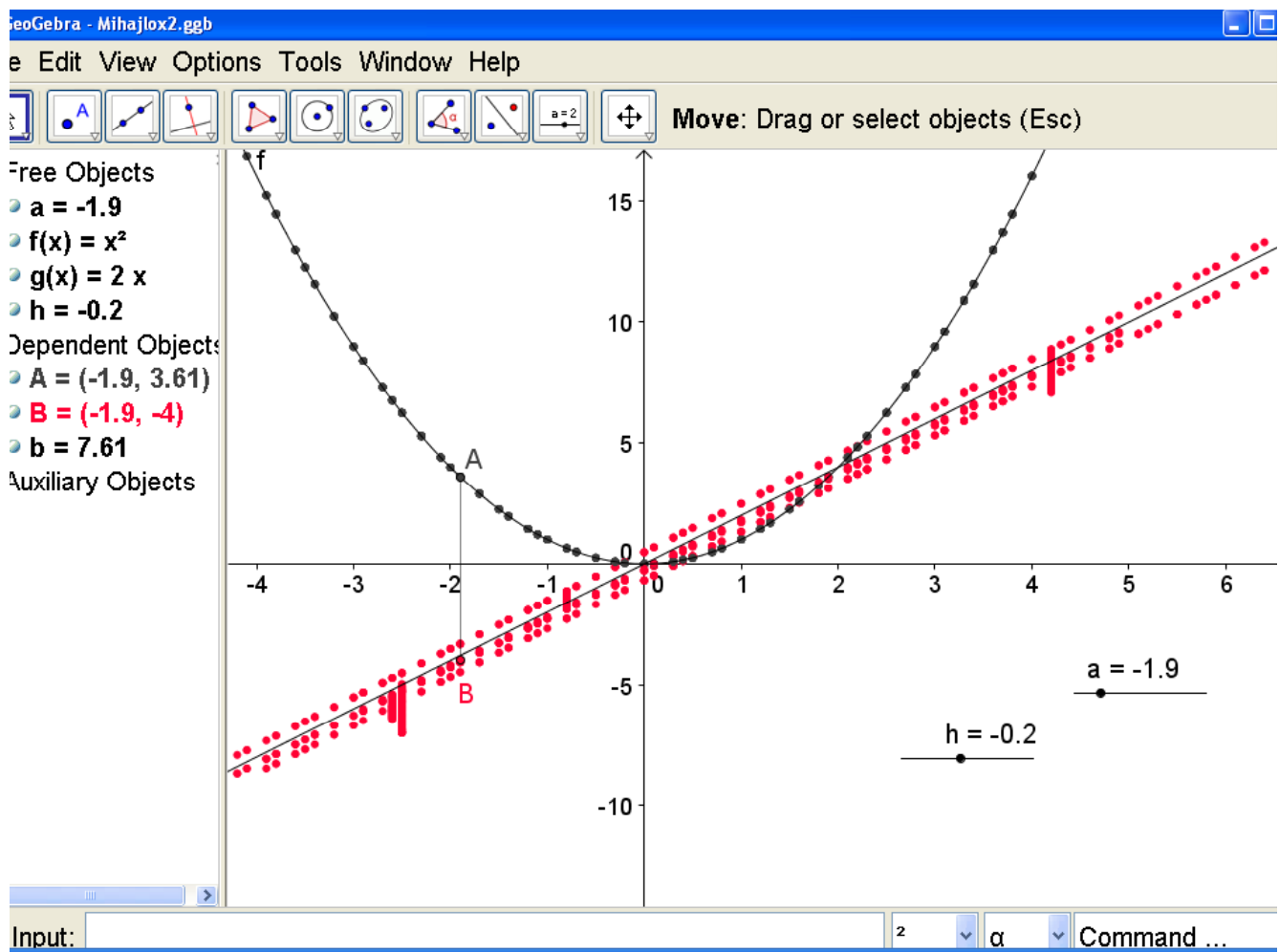
Geogebra still does work in three dimension.

Mihajlo start first, using sliders

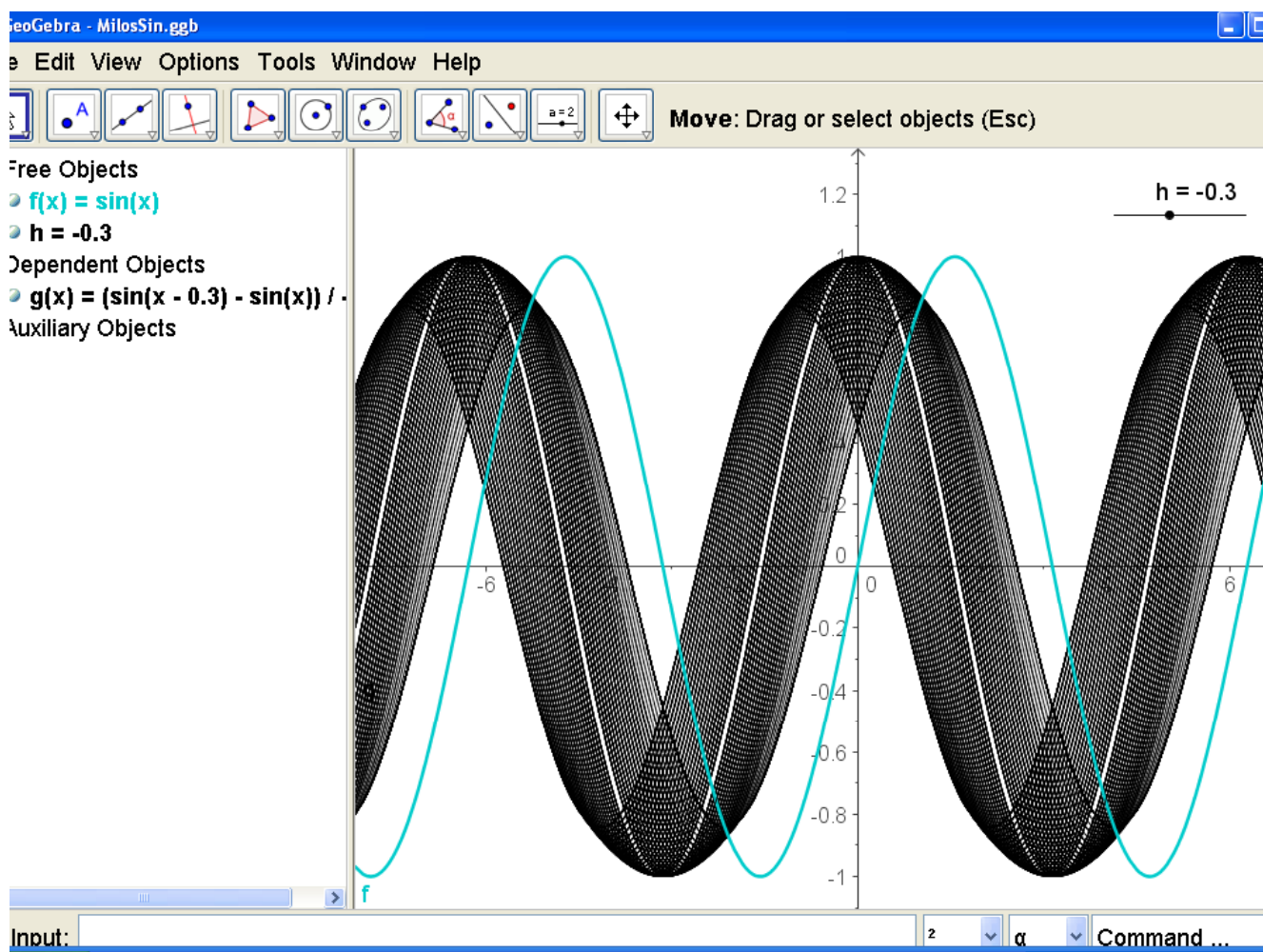
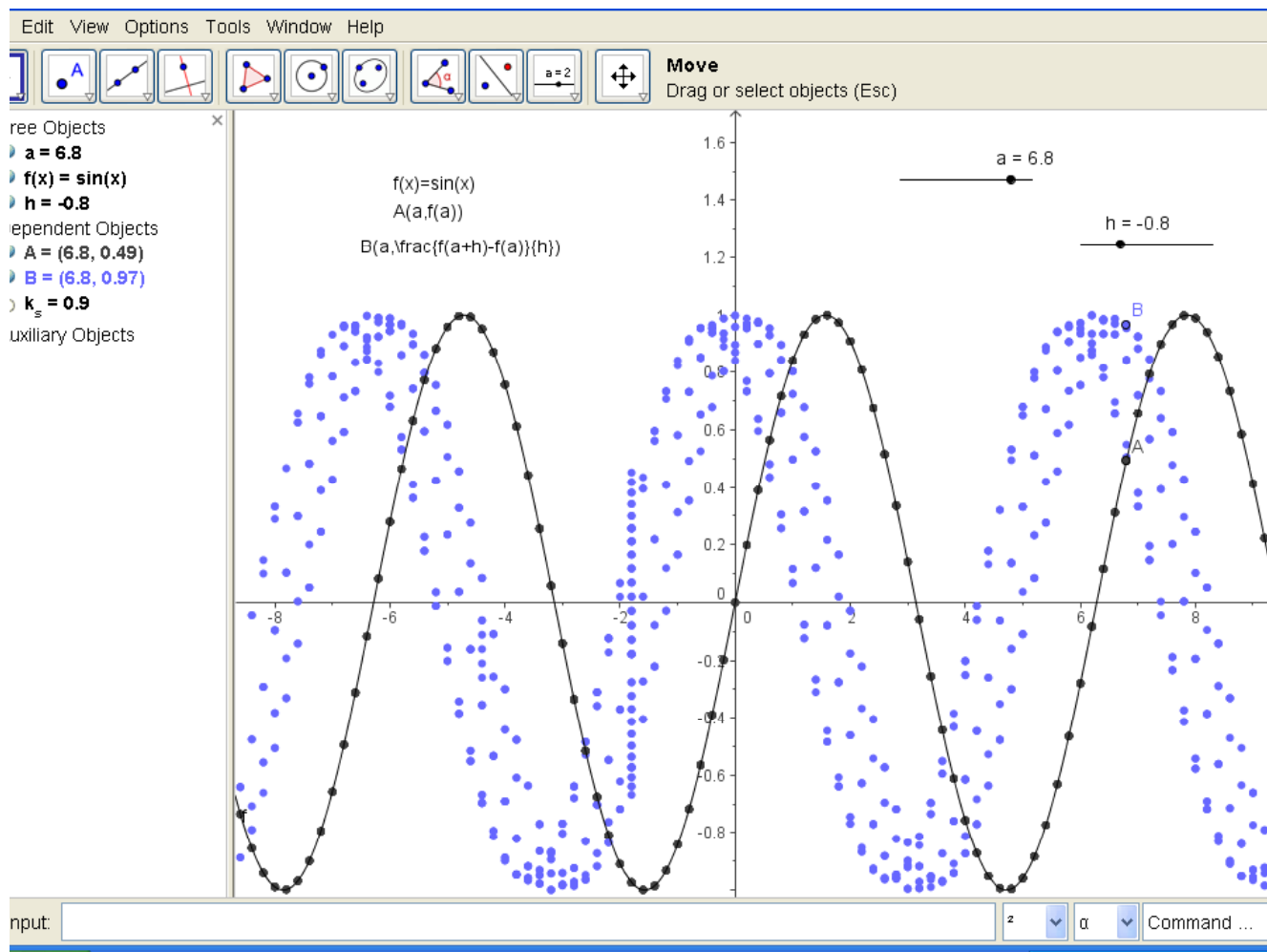
# On the visualization of the calculus concept

We choose the most interesting students work:

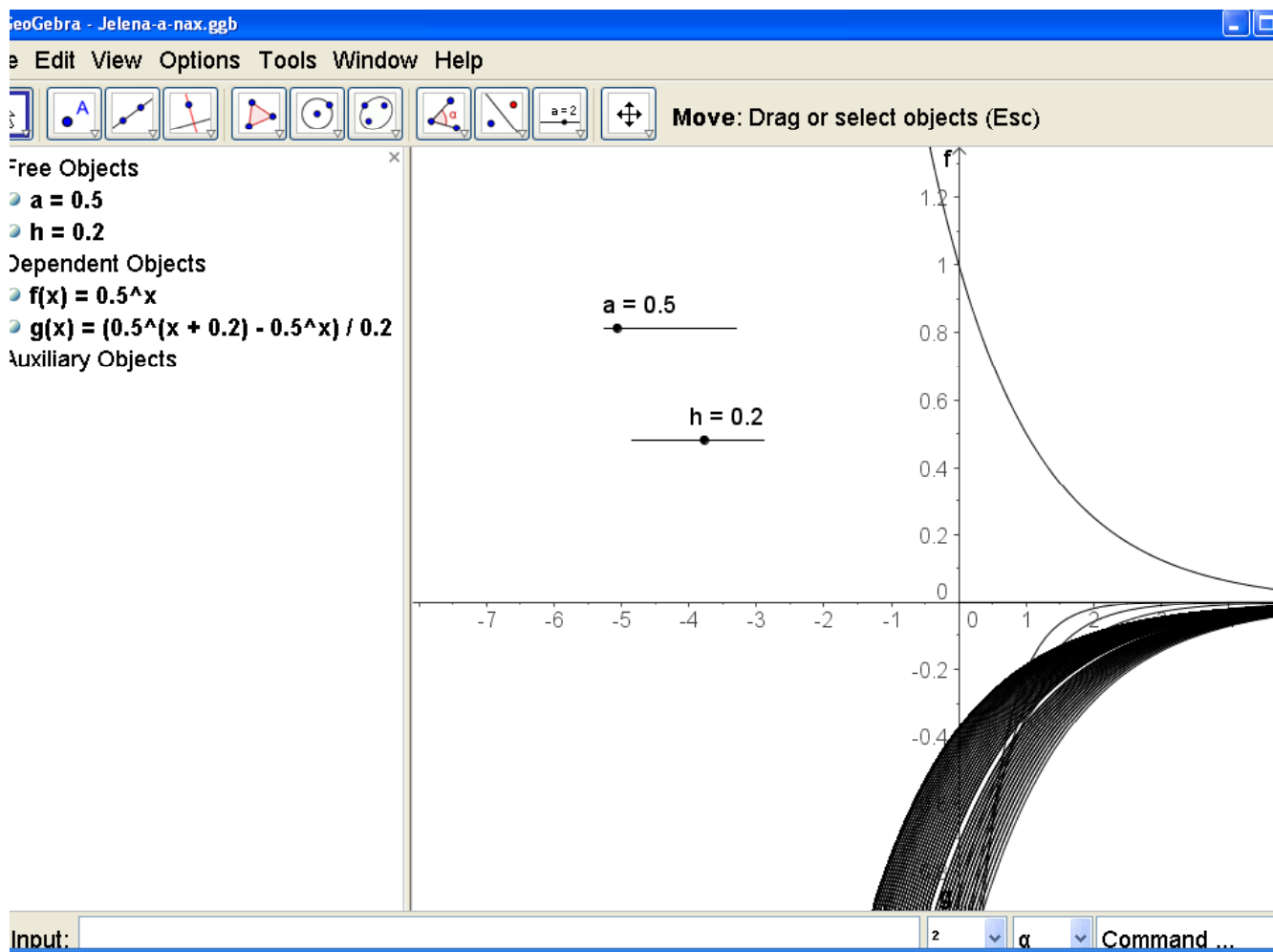
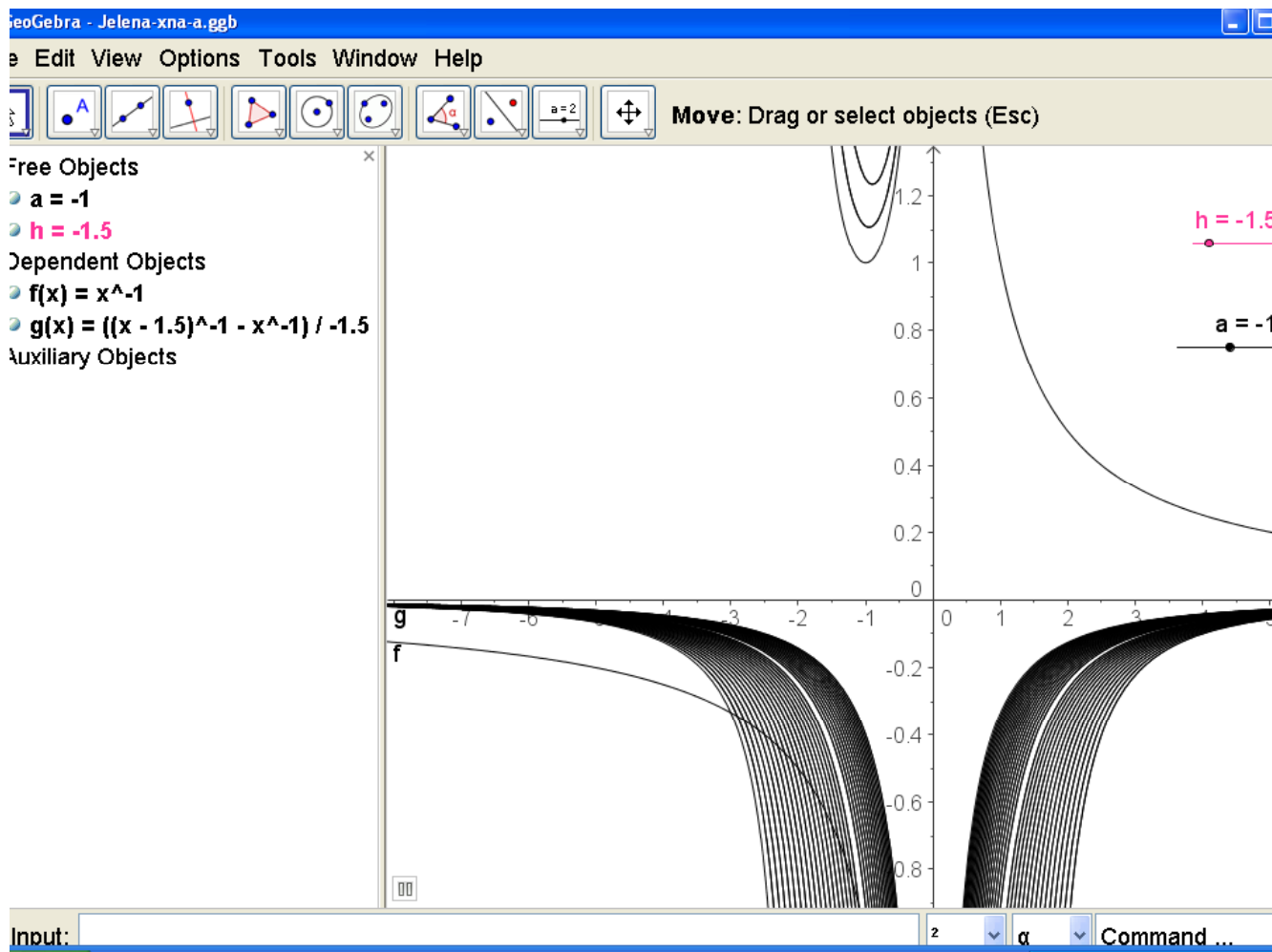
- [Mihajlox2.ggb](#)
- [ArpadSin.ggb](#)
- [MilosSin.ggb](#)
- [Jelena-xna-a.ggb](#)
- [Jelena-a-nax.ggb](#)

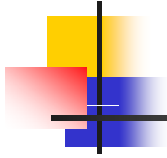












- The students used sliders in  $h$  order to make visualization of the first derivative at the point,
- and slider  $a$  order to make visualization of the first derivative function.



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■ Thank you for your attention

