See the Following papers for background to this slide presentation:

Oates, G. (2011). Sustaining integrated technology in undergraduate mathematics. *iJMEST Special Edition* (In print).

Oates, G. (2010). Integrated Technology in Undergraduate Mathematics: Issues of Assessment. *Electronic Journal of Mathematics and Technology, 4(2)*, pp.162-174. Available at http://www.radford.edu/ejmt

Oates, G. (2009). Relative values of curriculum topics in undergraduate mathematics in an integrated technology environment. *In R. Hunter, B. Bicknell & T. Burgess, (Eds.), Proceedings of the 32nd Annual Conference of the Mathematics Education Group of Australasia, Vol 2, Palmerston North, New Zealand: MERGA,* pp. 419-427.

"There is nothing either good or bad,
but thinking makes it so."

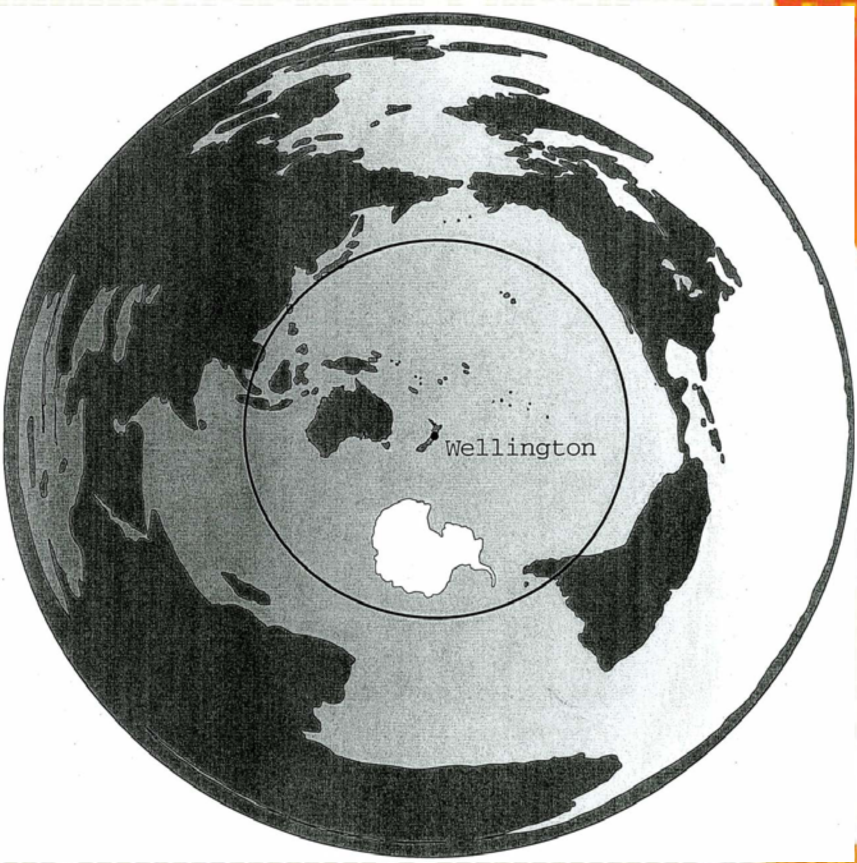
Shakespeare, 1601
Hamlet (II.2.1)

"Calculators and computers are neither good
nor bad teaching tools,
only using them makes them either"

Kutzler, 2000b, p.11

Critical Issues of Technology Use In Undergraduate Mathematics: A Case Study of CAS

Greg Oates,
The University of Auckland



Research Questions

- **What are the characteristics of a tertiary *Integrated Technology Mathematics Curriculum (ITMC)*, and how might we measure the nature of such technology integration?**
- **How can we facilitate the effective and sustainable implementation of a tertiary *Integrated Technology Mathematics Curriculum?***

Final Survey: Technology Use in Tertiary Institutions (2007)

Part A Department, Course and Technology Details (3 questions).

Part B Nature of Technology Usage (14 questions, e.g.)

- Does your department have a global or consistent policy or guidelines with respect to technology use in its courses, or is this the decision of individual courses or teaching staff?
- Is technology permitted, required or not allowed in assignments, tests or examinations? Please specify, e.g. allowed or required in assignments; not used in tests/exams as not in laboratory; can use in exam (e.g. CAS or graphics calculator); question in exam that requires knowledge of technology (e.g. Matlab printout); tests or exams held in laboratory.
- Have there been any changes to the content of the course because of the technology you use? Please specify, e.g. have some topics been dropped? Have any new topics been added? Has the order of topics been changed?

Part C: Beliefs about technology use (e.g.)

- Please state your personal experience and your feelings and/or beliefs about the role of technology in undergraduate mathematics. Consider any benefits or disadvantages that you are aware of.

Part D: General (e.g.)

D1 What barriers/difficulties have you encountered or observed with regard to technology use and/or implementation in your course(s)?

D2 What changes would you like to see with respect to the use of technology in your course(s)?

D3 What changes do you anticipate in your course(s) in future because of technological developments

C2	Please select the response that best represents your view about the ideal use of calculators and computers.	Number of Responses (1= Little or No Emphasis, 5 = Heavy)					
		1	2	3	4	5	Mean
(i)	Calculators for numerical or computational purposes	4	1	7	7	9	3.5
(ii)	Calculators for graphing purposes	5	5	4	9	6	3.2
(iii)	Calculators for symbolic manipulation	11	6	5	4	3	2.4
(iv)	Computer software (e.g. Matlab, Maple)	1	1	6	13	8	3.9
(v)	Modifying existing software or programming	4	8	4	5	6	3.2
(vi)	Spreadsheets or tables	4	6	4	7	5	3.1

Table 5.7: A Refined Taxonomy for Integrated Technology

Taxonomy Component	Sub-Category	Literature References and Sample Survey Responses. (Page numbers for location in this study, not to cited reference)
Access	• Choice of Technology: Portability and Access outside of class- time? Calculators versus computers? • Assessment: Allowed to use? Congruency? • Cost and equity: Related to choice. • Staff Access: Shared? In classrooms?	Pp. 119-127: Zevenbergen, 1999, 2001; Bradley & Kemp, 2002; Van der Hoff & Harding, 2007. p. 137: Heid, 1997; Leigh-Lancaster, 2000; Forgasz, Griffith & Tan, 2006. "It has many benefits if all the students can reach almost the same technology, otherwise it creates important differences between them. I would like to see all my students using laptops, as in the private universities." G6, Uruguay. "The lab space is limited so only so many classes in one semester can have access to software available in the lab." H6, Australia.
Assessment	• Congruency: Classes, Assignments, Homework, Tests, Exams? • Technology Assumed? Neutral? Active? Free? Prohibited? Question setting: Level of difficulty, PTK. • Multiple Formats: Computer-based Assignments? On-line testing, submission? Computer-Aided Testing? • Different Student Solutions.	Pp. 138-141: Rochowicz, 1996; Stephens & Leigh-Lancaster, 1997; Hong, Thomas & Kiernan, 2000; Stacey, Asp & McCrae, 2000; Forster & Mueller, 2002; Garner & Leigh-Lancaster, 2003: Engelbrecht & Harding, 2003; 2004; Ball & Stacey, 2004. Pp. 54, 73, 78: Leigh-Lancaster, 2000; Anguelov, Engelbrecht & Harding, 2001; Stewart, Thomas & Hannah, 2005. Pp. 69- 70: Stacey, Asp & McCrae, (2000); Hong & Thomas, 2006. "The traditional three-hour written exam has been with us for decades if not centuries. Computers will become increasingly better for the testing of key mastery skills in all subjects. They are complementary…A computerised formal test or exam can impose an additional stress on students and must only be introduced when students are fully experienced in its use." G4, United Kingdom. "Students may use any hand held calculator, but in exams they must show full written working to reach the answer. Calculators are often used to check results". N6, Australia.
Organisational Factors	• Planning: Technology Policy and Goals; Consistency versus Staff Autonomy. • Course Design: Syllabi; Delivery and Access, e.g. labs, large lectures, Datashows. • Budget: Technology Choice; Licensing. • Service Course Relationships. • Sustainability: Professional Development; Evaluation; Consistent Policy. Time for change, training, resource preparation.	Pp. 47-52: Smith, 1998; Stacey, Asp & McCrae, 2000; Hillel, 2001a; Holton, 2005; Pierce, Turville, & Giri, 2003. Pp. 119-133: Brown, 1996; Ganter, 1999; Galbraith & Pemberton, 2000; Engelbrecht & Harding, 2001; Keynes & Olson, 2001; Muller, 2001; Buteau & Muller, 2006; Davis, Porta & Uhl, 2006; Bonnington, Oates, Parnell et al., 2007; Thomas & Chinnappan, 2008. "The technology has to be carefully planned so that it does not create more difficulties than it is supposed to solve." H6, Australia. "It depends on the objectives of the subject. In some technology would be more integrated than in others." H13, New Zealand. "Bureaucracy slow to change. Use often isolated to single course." N7, South Africa.

Table 5.7 continued: A Refined Taxonomy for Integrated Technology

Taxonomy Component	Sub-Category	Literature References and Sample Survey Responses.
Mathematical Factors	• Content: Order and value of topics. • Subject Imperatives: e.g. Algebra-CAS & symbolic manipulation; Service, Applied, Pure courses; Domain-specific technology. • Cognition, Reasoning and Skills: Technical versus Conceptual, IA and MK; Representational Versatility; APOS theory; Objects & Procepts; Design Limitations. • Mathematical Knowledge: Nature of Mathematics, Objectives and Goals; Needs of users verus learners.	Pp. 52-69: Smith, 1998; Artigue, 2000; 2002; Stacey, Asp & McCrae, 2000; Tall et al., 2000; Herget et al., 2000; Harman. 2003; Heid, 2003; Kutzler, 2003; Stacey, 2003; Holton, 2005. Pp. 88-100: Hillel, 1993; Noss & Hoyles, 1996; Lagrange, 1999; Tall, 1999-2002; Doerr, 2001; Dubinsky & McDonald, 2002; Dana-Picard & Kidron, 2006; Lynch, 2006. Pp. 108-127: Love, 1995; Hong & Thomas, 1997; 1998; Wester, 1999; Bloom et al., 2001; Hillel, 2001; Thomas, 2001; 2007; Stewart & Thomas, 2003; Laborde et al., 2006. "Less emphasis on techniques, more powerful visualisation." H14, New Zealand. "For us, courses drive technology rather than technology driving courses. Technology is seen as a tool for accomplishing teaching objectives." H12, USA. "The big problem is how much in the way of skills do students need to have and how much is OK if they rely on technology to do algebraic manipulation etc." G3, New Zealand.
Staff Factors	• Type of Use: Professional Domain; Modelling Technology; Teacher Privileging; Applications and/or Educational. • Proficiency: Instrumental Genesis; PTK; Affordances & Constraints. Interactions. • Beliefs and Attitudes: Nature of Maths; Technology; Learning; Constructivism. • Training & Support: Professional Development, Consistency & Sustainability. • Time: Change, Resource Preparation.	Pp. 69, 81-82: Hong & Thomas, 2006; Thomas & Chinnappan, 2008. Pp. 89-110: Kendal & Stacey, 1999; 2001; Chinnappan & Thomas, 2000; Devries, 2000; Goos et al., 2000; Brown et al., 2004; Pierce & Stacey, 2004; Artigue, 2006; Laborde et al., 2006. Pp. 121-136. Love, 1995; Cretchley et al. 1999; Hong, Thomas & Kiernan, 2000; Anguelov et al., 2001; Keynes & Olson, 2001; Muller, 2001; Norton & Cooper, 2001b; Kersaint et al., 2003; Zbiek, 2003; Buteau & Muller, 2006. "Some staff insist that the students still complete these calculations by hand." H6, Australia. "Some staff believe students will lose ability to do routine calculations." G3, New Zealand. "Technology should be integrated only by staff who believe it is useful. Imposition of technology seems to have a negative effect on all involved." G7, Australia.
Student Factors	• Type of Use: Technical, Functional? • Proficiency: Instrumental Genesis; Mathematical Reasoning; Solutions. • Training & Assistance: Tutorials, Manuals. • Personal: Previous experience; Motivation & Needs, e.g. major, service course; Technology ownership, Equity. • Beliefs & Attitudes: Teacher Privileging.	Pp. 48-50: Schwartz, 1999; Hillel, 2001b; Steen, 2001; Holton, 2005. Pp. 97-115: Guin & Trouche, 1999; Tall, 1999-2002; Goos et al., 2000; Hong, Thomas & Kwong, 2000; Norton et al, 2001; Artigue, 2002; 2006; Heid, 2003; Lagrange et al., 2003; Goos & Cretchley, 2004; Hoyles et al., 2004; Pierce & Stacey, 2004; Thomas, Monaghan & Pierce, 2004; Stewart et al., 2005; Andresen, 2006; Van der Hoff & Harding, 2007. "It's difficult (for students) to make sense of the use of technology, especially those who had High School maths teachers with strong opinions against the use of technology." H2, Canada. "A more positive attitude from students as to what it can contribute to their learning and applications of mathematics." H13, Australia.

A Taxonomy for Integrated Technology

Taxonomy Component	Characteristic Survey Response for Taxonomy Component
Access	"It has many benefits if all the students can reach almost the same technology; otherwise it creates important differences between them. I would like to see all my students using laptops, as in the private universities." (Uruguay)
Assessment	"Students may use any hand held calculator, but in exams they must show full written working to reach the answer. Calculators are often used to check results". (Australia)
Organisational Factors	"Bureaucracy slow to change. Use often isolated to single course." (South Africa)
Mathematical Factors	"Less emphasis on techniques, more powerful visualisation." (New Zealand)
Staff Factors	"Technology should be integrated only by staff who believe it is useful. Imposition of technology seems to have a negative effect on all involved." (Australia)
Student Factors	"It's difficult (for students) to make sense of the use of technology, especially those who had High School maths teachers with strong opinions against the use of technology." (Canada)

Staff Factors:

- Type of Use: Professional Domain; Modelling Technology; Teacher Privileging; Applications and/or Educational.
- Proficiency: Instrumental Genesis; PTK; Affordances & Constraints. Interactions.
- Beliefs and Attitudes: Nature of Maths; Technology; Learning; Constructivism.
- Training & Support: Professional Development, Consistency & Sustainability.
- Time: Change, Resource Preparation.

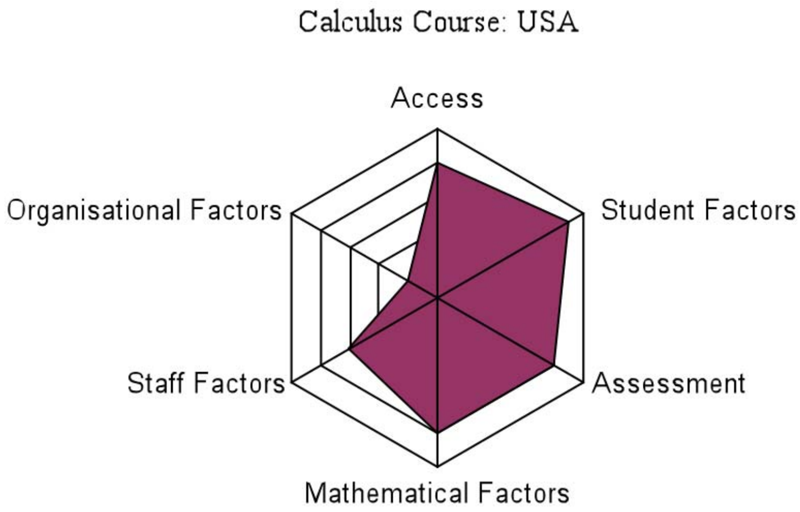
Assessment

- Congruency: Classes, Assignments, Homework, Tests, Exams?
- Technology Assumed? Neutral? Active? Free? Prohibited? Question setting: Level of difficulty, PTK.
- Multiple Formats: Computer-based Assignments? On-line testing, submission? Computer-Aided Testing?
- Different Student Solutions.

Mathematical Factors

- Content: Order and value of topics.
- Subject Imperatives: e.g. Algebra-CAS & symbolic manipulation; Service, Applied, Pure courses; Domain-specific technology.
- Cognition, Reasoning and Skills: Technical versus Conceptual, IA and MK; Representational Versatility; APOS theory; Objects & Procepts; Design Limitations.
- Mathematical Knowledge: Nature of Mathematics, Objectives and Goals; Needs of users verus learners.

Calculus course from the United States showing gap in technology integration for "Organisational Factors".



Conclusion from Oates, 2009, p. 251

The results from the observational study and the interviews suggest that technology implementation must recognise the inter-related structure of the taxonomy... Attendance to the factors in a comprehensive fashion results in higher and more sustainable levels of technology integration... Attendance to some elements in isolation may stimulate changes, but is unlikely to lead to sustained and effective technology integration.

Comparison of Maths 108 Examination Questions from 1999 to 2007

Year	Section of Exam: marks/total	Percentage of CAS-Positive Mark in this section.		
		TI-89	TI-92	Undecided
1999 (Before CAS-calculators)	Short Answers: 30/100	80	0	0
	Long Answers: 70/100	33	39	0
2004 (CAS-calculators)	Multiple Choice: 54/180	26	37	4
	Long Answers: 126/180	42	56	6
2007 (Matlab as principal technology, CAS-calculators allowed)	Multiple Choice: 40/120	35	0	0
	Long Answers: 80/120	40	0	8

Example and Year	Question
Example 1: 1999	If $f(x)=\frac{2x+5}{3x+1}$, find $f'(x)$
Example 2: 2007	When differentiating the following functions, for which is the Chain Rule useful? (a) $f_1(x)=\tan x\cdot\ln x$ (c) $f_3(x)=\tan x(\ln x)$ (b) $f_2(x)=\frac{\tan x}{\ln x}$ (d) $f_4(x)=e^x\tan x$
Example 3: 2007	Suppose it is known that $\int f(x)dx=e^x+C$. Then $\int f(x-1)dx=$ (a) e^x+C (b) $e^{x-1}+C$ (c) e^x-1+C (d) $e^x(x-1)+C$
Example 4: 2004	The function f, where $f(x)=\ln(\ln(x))$ has domain: (a) $(0,1)$ (b) $(-\infty,0)$ (c) $(0,\infty)$ (d) $(1,\infty)$
Example 5: 2004	Given that x and y satisfy the equation $x^2-y^2=2xy+1$. One takes differentials. Which of the following is true? (a) The result is $2x-2y(dx+dy)=2$. (b) It is not possible to take differentials in this case. (c) The result is $2xdx-2ydy=2ydx+2xdy$. (d) The result is $2dx-2dy=2dxdy$.
Example 6: 2007	Let $A=\begin{bmatrix}1&3\\4&2\end{bmatrix}$ and $C=\begin{bmatrix}4&4\\6&6\end{bmatrix}$. If $AB=C$, then which of the following represents the matrix B? (a) $\begin{bmatrix}4&4\\0&0\end{bmatrix}$ (b) $\begin{bmatrix}1&0\\0&1\end{bmatrix}$ (c) $\begin{bmatrix}-2&-2\\2&2\end{bmatrix}$ (d) $\begin{bmatrix}1&1\\1&1\end{bmatrix}$
Example 7: 2007	$\int 12x^3\,dx=$ (a) $3x^2+C$ (b) $4x^2+C$ (c) $3x^4+C$ (d) $4x^4+C$

Assessing Technological Advantages: Long Answer question.

The matrices $A = \begin{bmatrix} 1 & 0 & 1 \\ -1 & 1 & 1 \\ 3 & -1 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 1 & 1 \\ -3 & 3 & 2 \\ 2 & -1 & -1 \end{bmatrix}$ are inverses of each other.

Use this fact to solve the following system of linear equations:

$$\begin{aligned} -x + y + z &= 4 \\ -3x + 3y + 2z &= 4 \\ 2x - y - z &= 1 \end{aligned}$$

Instructor's PTK (Pedagogical Technical Knowledge)

"necessary knowledge of the principles and techniques required to teach mathematics using a given technology "

IMPLICATIONS

- Nature of assessment itself is not examined, sometimes even used as an argument against technology (i.e. Technology-neutral or technology -free exams)
- Many tertiary staff lack sufficient PTK to design effective assessment strategies or evaluate exam items

In latter Auckland case (Matlab in computer labs), exam questions don't actively involve technology. Get questions such as:

26. Which one of the following is a useful Matlab command for sketching an equation?
(a)ezplot (b) drawit (c) plotit (d) ezdraw

"notwithstanding the holistic consideration of the taxonomy as advocated by Oates (2009), assessment issues remained a significant individual factor in technology implementation at The University of Auckland. The impact of CAS on examination questions is seen as a particularly complex issue. Questions require real constant care and attention to balance the examination of students' skills against conceptual understanding in a fair and appropriate manner.

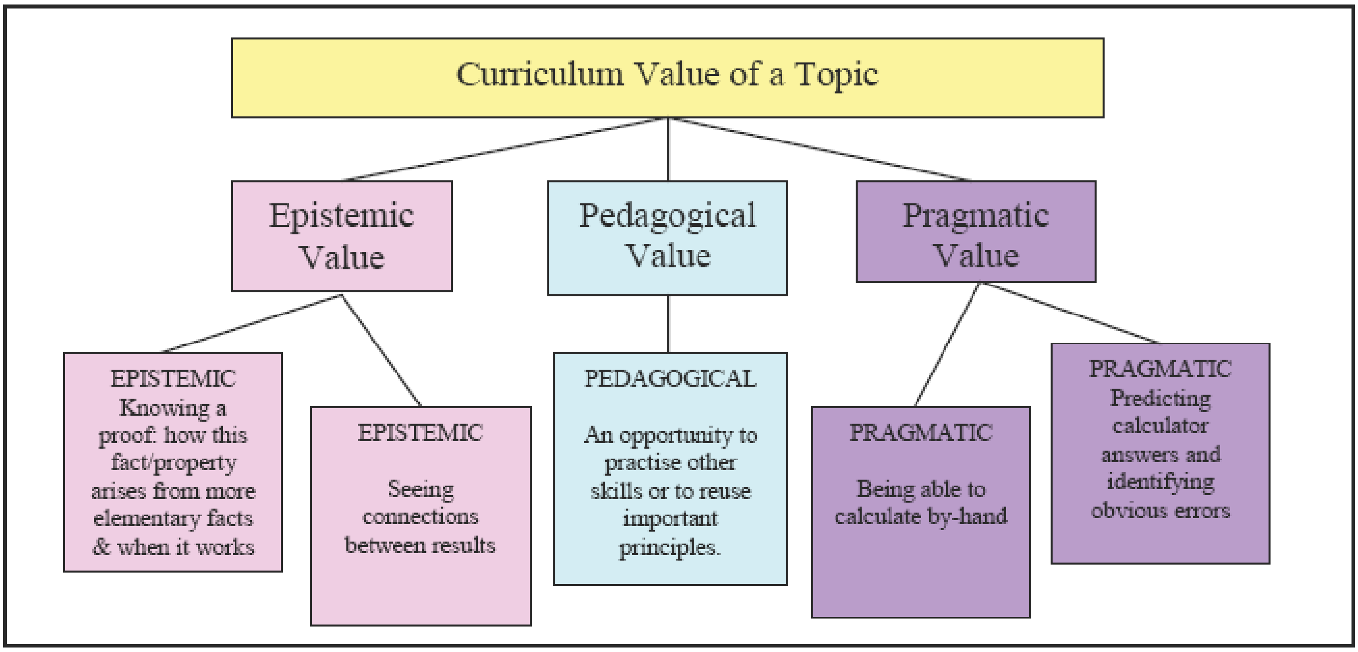
"assessment issues remain problematic, even in an otherwise integrated environment...Continued vigilance is required to attend to the inequitable advantages afforded by unequal access to technology", whether that be physically, through differing levels of student instrumentation, or the affordances provided by different types of technology (Oates, 2009, p. 253)

...what most students learn in their mathematics courses is, to carry out a large number of standardised procedures, cast in precisely defined formalisms, for obtaining answers to clearly delimited classes of exercises.

They end up with a considerable amount of mathematics knowledge but without the working methodology of the mathematician,

(Dreyfus,1991, p. 28)

The curriculum value of topics is markedly changed by the introduction of CAS. Old justifications for teaching topics, especially pragmatic justifications, will not necessarily apply...The educational community needs to build up sophisticated rationales for curriculum areas that were not debated in the past. Justifications may be on pragmatic, epistemic, or pedagogical grounds. (Stacey, 2003, p. 7)



Topics can have epistemic, pragmatic and pedagogical value. (Stacey, 2003).

1. (It's a) bad idea to teach this obsolete, tedious *LU* factorisation, which no one needs anymore…(while) it still has some applicability, … currently no one client department needs it…we should give preference to teaching ideas, not techniques.

2. The issues divide neatly between the importance of the technique and the extent to which it should be laboured as a teaching item. The topic is intrinsically important because it is at the centre of all practical scientific computation…Is it important as a teaching item? Sometimes a mathematical concept has to be introduced without a directly practical application…I personally think that the opportunity to introduce *LU* with pivoting is like finding a flower in the desert.

3. This flower unfortunately has to be uprooted, together with integration by trigonometric substitution and other techniques which have lost relevance for the wider audience. A regular person with regular needs will be much better off using Maple. Such techniques should be taught in specialised courses…It takes too much precious time which can be better spent on building understanding.

4. I believe that a good portion of (any) honest technique is useful for students learning mathematics, as a training of ability for prolonged logical concentration. Separating learning of ideas from learning of adequate technical support looks similar to learning by heart a French song without learning French language.

Before computers, there used to be a big emphasis on special techniques for special differential equations, …students had to recognise some 15 different types of differential equation, you had no options, you had to solve it explicitly, there was no numerical option. You had to know the technique, all that's gone, if you don't recognise a differential equation, you whack it on a computer.

Curve sketching will be relegated to tutorials and exercises, supported with CAS. In using CAS for curve sketching, it is particularly important to emphasise the need to consider the critical points to get an idea of what interval of the domain to graph the function over.

Most students can barely see how it fits, but they get used to a standard technique of putting the things in, … the question mathematicians need to answer now is, do such mechanistic techniques generalise to more general problem-solving type situations later which are going to be useful?, and I think the answer is no.

Grasping the relation between elementary row operations and equivalent systems is the key notion, not the actual procedure for row-reducing matrices. Once understood, I see no reason why students should not be given free rein to use CAS and go directly to row reduced echelon form of a matrix without actually performing the row operations (and later, to go directly from a system of equations to a CAS-generated solution). (Hillel, 2001b, p. 374)

The main problem doesn't seem to be that they can't do the operations (for which the calculator can help them); it's that they don't know what operations to do. They'll do three pages of working and still won't have any zeros in their matrix…the students don't understand what the goal is. I'm not sure how technology can help with this.

Depends what one wants, I can't see how a student can understand the process by pushing a button, it may be OK for an engineer who just needs a seriously good program to provide the numbers at the end, they don't need to know anything about Gaussian elimination, …but most students haven't got a clue what their answer means, they know nothing more about their solutions than that they are a result of what they do.

[IR 5] changed his mind several times as he thought through the issue... Initially he was quick to discount any pragmatic or epistemic value in teaching the procedure itself, comfortable that students could use technology to directly calculate row-reduced matrices, using an analogy of driving a car without knowing how it works. ...However, he later reconsidered this, and decided that students really do need to understand the process first, before using the black-box. (but)...like many colleagues and students, (he) enjoys Gaussian elimination as a process, “there’s an intrinsic enjoyment that makes it worth doing”

When you talk about epistemological and pragmatic, there’s another kind of question I think mathematics is really about, and that is training the human mind. If you take Gaussian elimination, there is an argument for teaching this, since if students don’t master (such topics), they don’t even develop mathematical structures that are relevant for making mathematical assessments about any other problems they meet. [IR 1]

“Technology integrated intelligently with curriculum and pedagogy produces measurable learning gains” (Tall, Smith & Piez, 2008).

Content issues were a significant factor in the technology implementation at The University of Auckland. Reaching consensus on the relative values of topics in the undergraduate mathematics curriculum was especially problematic...

A re-examination of the changing pragmatic and epistemic values of specific topics, and the goals of mathematics education, within a rapidly evolving technological environment, remains a pressing challenge for undergraduate mathematics educators.”
(Oates, 2009)