Categorizing CAS Use within One Reform-Oriented

United States Mathematics Textbook

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Lecture Proposal for the ACDCA Strand

ABSTRACT

This study introduces a theoretical framework that can be used to examine the use of computer algebra systems in written curricula. This framework was used to examine CAS activities and student textbook problems in one US reform oriented mathematics textbook intended for students, aged 16-17. The role of CAS in promoting reasoning and proof was also examined. CAS was deployed principally by the textbook authors in an active fashion and it was the primary means of solving problems when it appeared in the textbook. CAS was used primarily as a tool to generate equivalent equations and rarely to experiment with function parameters. The majority of the CAS results were non-formulaic in nature and although the technology can produce purely abstract forms, this *aspect of the technology was rarely used. CAS was used most frequently to assist students in identifying patterns within the areas of reasoning and proof. Comparisons between the current third edition of the student textbook and the second edition suggest that the CAS integration resulted in a pedagogical change in the curriculum by making activities more investigative.*

A number of national documents in the United States (Common Core State Standards Initiative, 2010; National Council of Teachers of Mathematics, 2000) and other countries (Lew, 2008) highlight the importance of technology in helping students to learn mathematics. Graphing calculator use has spread in the United States, but computer algebra systems (CAS) have not. For instance, only 15% of teachers in the United States reported using this technology in a national survey (Braswell et al., 2001).

There are a number of possible reasons why this use of computer algebra systems is so low. These include the procedural nature of school mathematics in the United States (Fey, 1979; Jacobs et al., 2006; National Advisory Committee on Mathematical Education, 1975; Stigler & Hiebert, 1999), teachers' beliefs that teaching is telling (Chazan & Ball, 1999; Cooney, 1999), the limited use of CAS in US high school mathematics textbooks, and external high stakes exams such as the SAT and the ACT which do not allow students to use handheld calculators equipped with computer algebra systems.

The United States contains several very different educational systems. The majority of states in the US allow individual school districts (which contain several schools over a variety of geographic areas) to determine which textbooks students will use. The remaining states contain committees that review textbooks and determine which ones are suitable for students. A list of approved texts in an area such as high school mathematics are developed by this committee and individual districts or schools can choose from these textbooks with the state typically covering the cost of purchasing textbooks.

Since large states such as California, Texas, and Florida use state textbook adoption committees to determine suitable textbooks there is a strong effort by publishers to tailor their products towards these curriculum committees. Consequently, these states help to drive the textbook market in the United States (Keith, 1991). In 1989, the National Council of Teachers of Mathematics (NCTM) developed the *Curriculum and Evaluation Standards for School Mathematics* (hereafter referred to as the *Standards*). This document made a number of different recommendations for the development of elementary, middle, or high school mathematics curricula. However, given the textbook market in the United States and the radical vision proposed in the *Standards*, the government knew that the only way that curricula aligned with this document would be created is through government intervention. Thus, during the 1990's the National Science Foundation (NSF) in the United States funded a total of 13 curricula at the elementary, middle, and high school levels.

These curricula have often been grouped together and given different labels: reform-oriented, *Standards*-based, or NSF-funded. Curricula that were not part of the NSF funding have frequently been described as conventional mathematics curricula (Stein, Remillard, & Smith, 2007). Reform-oriented curricula typically contain greater concentrations of the following categories than conventional curricula: technology, alternative representations, real-world contexts, and mathematical processes (e.g., communication).

One difference between the use of technology in conventional curricula and reform-oriented programs is captured in the following two figures. Figure 1 provides as example from a conventional curricula where graphing calculator technology is used to solve a mathematical problem by presenting students with an alternative representation, this case a graph, instead of solely a symbolic representation.

SOCCER **A goalie kicks a soccer ball with an upward velocity of 65 feet per second, and her foot meets the ball 1 foot off the ground. The quadratic function $h = -16t^2 + 65t + 1$ represents the height of the ball h in feet after t seconds. Approximately how long is the ball in the air?**

You need to find the roots of the equation $-16t^2 + 65t + 1 = 0$. Use a graphing calculator to graph the related function $f(x) = -16t^2 + 65t + 1$.

Figure 1. Use of technology to solve a mathematical problem from a conventional mathematics curriculum (Carter et al., 2010, p. 545).

In *Standards*-based curricula technology is used as a tool to solve mathematical problems, but it is also used to help students to learn mathematical concepts. For example, in Figure 2, taken from the *Core-Plus Mathematics* program (Hirsch et al., 2008, p. 474) students use graphical and tabular representations to understand the effect of a on the general quadratic function $y = ax^2$.

> (1) Study the tables and graphs produced by such functions for several *positive* values of a. For example, you might start by comparing tables and graphs of $y = x^2$, $y = 2x^2$, and $y = 0.5x^2$ for $-10 \leq x \leq 10$.
>
> **a.** What do all the graphs have in common? How about all the tables?
>
> **b.** How is the pattern in a table or graph of $y = ax^2$ related to the value of the coefficient a when $a > 0$?

Figure 2. Use of technology to develop a quadratic function concept within a reform-oriented curriculum.

When the *Standards* were developed many conventional curricula included the use of scientific or four-function calculators. Another common integration of technology at this time was the use of BASIC computer language programs to perform a variety of different mathematical functions. During the late 1980’s the University of Chicago School Mathematics Project (UCSMP) curriculum, designed for students aged 12 – 18, foreshadowed several of the recommendations of the *Standards*. In the area of technology, the curriculum integrated use of automatic graphers (i.e., graphing calculators) as seen in Figure 3.

Graph $y = 3x^2 + 4x - 2$ using an automatic grapher.

Solution 1

1. The equation is already solved for y. Key in
   y $\boxed{=}$ 3 $\boxed{\times}$ $\boxed{x^2}$ $\boxed{+}$ 4 $\boxed{\times}$ $\boxed{x}$ $\boxed{-}$ 2 for a calculator or
   y = 3*x*x + 4*x − 2 for a computer.
2. To determine a window, you can find some points on the graph by hand. To begin it is useful to pick values of x not so close to each other.

x	y
0	-2
10	338
5	93
-5	53
1	5
-1	-3

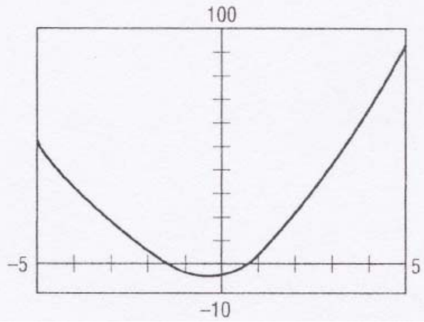

Figure 3. Early use of graphing calculators in an early reform-oriented mathematics textbook intended for students aged 16 to 17 (McConnell, 1990, p. 595).

Handheld CAS machines became widely available in 1995 with the advent of the TI-92 calculator. Since this was near the end of the development period for the first edition of *Standards*-based mathematics textbooks only one of these programs incorporated CAS technology: *Mathematics: Modeling Our World* (Garfunkel, Godbold, & Pollak, 2000). Screen shots of the CAS available on a TI-92 calculator appeared infrequently in the fourth course of this textbook series. One of these examples is shown in Figure 4.

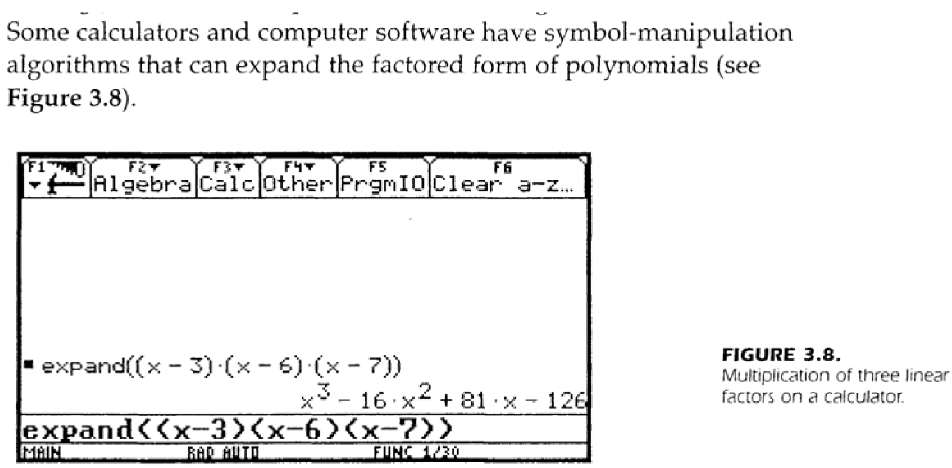
Some calculators and computer software have symbol-manipulation algorithms that can expand the factored form of polynomials (see **Figure 3.8**).

FIGURE 3.8.
Multiplication of three linear factors on a calculator.

Figure 4. Early CAS use in a reform-oriented mathematics textbook designed for students aged 17-18 (Garfunkel, Godbold, & Pollak, 2000, p. 199).

Despite this early use of CAS it appeared primarily as screenshots with some accompanying textual explanation. The authors did not expect students to possess this technology, thus this use of CAS was passive as opposed to active. That is, students were asked to read the examples, but they could not perform the same commands depicted in the screen shots on their own.

Within the last couple of years CAS has been integrated into three mathematics programs designed for students in the United States. These programs are *Core-Plus Mathematics* (Fey et al., 2009), *CME Project* (Education Development Center, 2009), and the third edition of the University of Chicago School Mathematics Project (Flanders et al., 2010). Since CAS has appeared only recently in mathematics textbooks in the United States, the purpose of this study is to present a framework for examining CAS use within the written curriculum, present the results of using this framework to note how

CAS is used in one secondary mathematics textbook, and describe how CAS is used in reasoning and proof in this textbook.

Methods

Curriculum

Although the University of Chicago School Mathematics Project (UCSMP) set of textbooks predated the *Standards* by a few years and received funding from private foundations it has been considered a *Standards*-based mathematics program (Senk & Thompson, 2003). This curriculum is unique in that it spanned six different years from age 12 up through the last year of high school – age 18. Most high school mathematics programs only included three to four years. The developers expected students to read this curriculum. In addition, it included the use of technology and real-world contexts to help students understand mathematical concepts. This textbook series used the acronym SPUR to recognize the importance of skills, properties, uses, and representations in its design. The teacher's edition of the *Advanced Algebra* (Flanders et al., 2010) textbook was the focus of this study as it was believed that content that was more advanced than the beginning algebra course would contain more frequent and varied CAS use. The *Advanced Algebra* textbook contains direct and indirect variation, matrices, power functions, inverses and radicals, trigonometry, polynomials, and series. The teacher's edition contains the student textbook as well as teacher resource materials in the outer margins and on separate pages.

Framework

Technology. This study began with an examination of different technology-based articles for frameworks that could be used in categorizing technology use within the written curricula. A constant comparison methodology (Glaser & Strauss, 1967) was used to refine the initial framework by using CAS examples from the textbook itself. One research article appeared to be particularly suited to the purposes of this study as it describes different roles that CAS can take on within mathematics curricula.

Heid and Edwards (2001) describe four different roles for CAS in mathematics curricula. First, they state that a CAS can be the “primary producer of symbolic results” (p. 130). As a result the curriculum could ask students to focus on other important mathematical processes such as translating a problem set within a realistic context to a mathematical setting by way of a formula or equation.

Second, the CAS could be used to produce symbolic procedures as seen in Figure 5. Here the CAS is used to perform the symbolic procedures associated with solving a linear equation leaving the user to focus on determining the steps needed to solve the equation or the concepts behind solving an equation. The CAS could also be used in this role to help students learn how to solve equations and detect efficient from inefficient procedural steps.

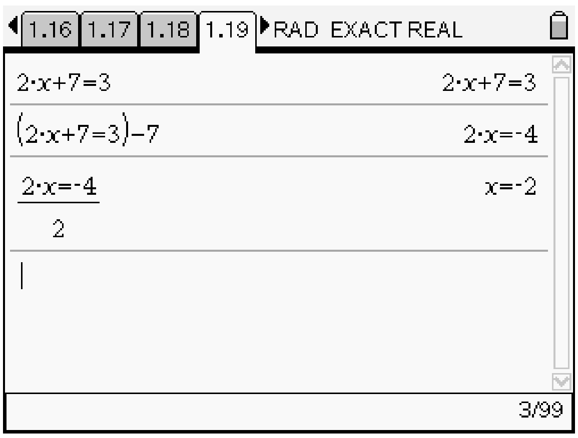

Figure 5. Using a CAS to solve a linear equation.

Third, a CAS can help students generate a number of different examples from which patterns in symbolic forms can be detected. An example of this use of the CAS is seen in Figure 6. Moreover, this use of the CAS fits nicely with what some describe as what is really all about, identifying patterns (Steen, 1990).

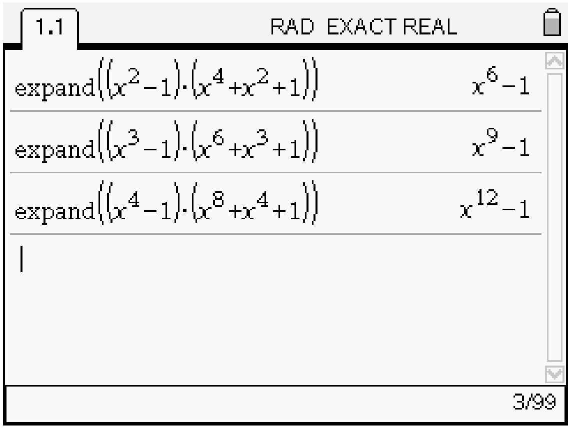

Figure 6. Using a CAS to multiply binomials and trinomials of particular forms from which to identify a symbolic pattern.

Fourth, the CAS can be used to produce symbolic formulas as seen in Figure 7.

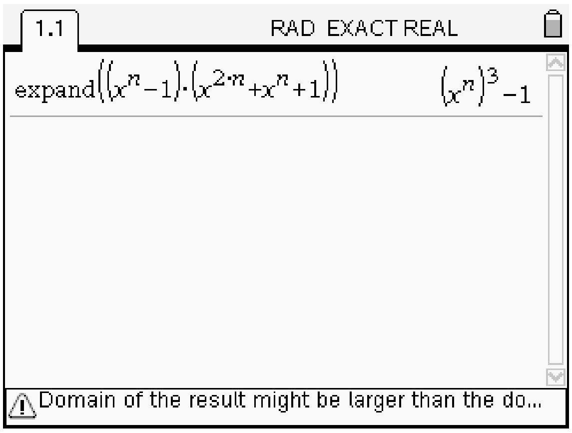

Figure 7. Using the CAS to manipulate symbolic forms that are entirely abstract or represent a general case.

In this case, students could be given a measure of independence to create their own formulas or work with completely abstract forms instead of relying on the textbook to provide them with this information.

These roles as well as how CAS was actually used in different reform-oriented mathematics textbooks led to the framework used in this study. The UCSMP *Advanced Algebra* (Flanders et al., 2010) student textbook provides completed examples for students to read as well as activities that students are expected to complete. This led to characterizing the CAS Interaction Level as either *passive* or *active*. Figure 8 depicts a passive use of CAS as students are simply asked to read through this example, not engage in using CAS themselves.

QuickTime™ and a
decompressor
are needed to see this picture.

Figure 8. Passive use of CAS in the *Advanced Algebra* (Flanders et al., 2010, p. 23) textbook.

An active use of CAS appears in Figure 9. This is a portion of an activity that students are asked to complete as part of their reading of a textbook the night before class is to take place or during the classroom lesson. It is active since students are asked to use their own CAS machines to evaluate a function at a specific value.

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are needed to see this picture.

Figure 9. Active use of CAS in the student textbook (Flanders et al., 2010, p. 22).

Examinations of the UCSMP textbook at the center of this study also revealed that CAS results could be isolated from other student work or connected. In the first case, students were simply asked to execute some command on the CAS such as factoring the expression $x^2 - 1$ and nothing more. An example of this is seen in Figure 10 as students are simply asked to evaluate a function at a specific value with the use of a CAS.

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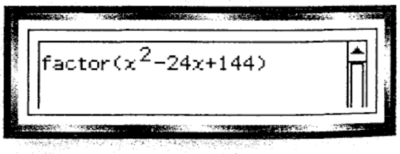
Figure 10. Isolated use of a CAS result (Flanders et al., 2010, p. 22).

In the second case, the visible result or the invisible procedure that the CAS used to produce it could be an object of reflection by the user. Another possibility is that the CAS result could be used by the individual in another task either with or without the use of technology. In both of these possibilities, the CAS result is connected. In Figure 11, students are asked to use CAS factorizations to describe how to factor different polynomial expressions.



MATERIALS CAS
Work with a partner.

Step 1 Factor the following expressions on a CAS.

a. $x^2 - 24x + 144$ b. $x^2 - 49$
c. $x^2 - 64$ d. $x^2 + 8x + 16$
e. $4x^2 - 81$ f. $x^2 - 6x + 9$
g. $x^2 + 10x + 25$ h. $36x^2 - 1$

Step 2 Based on the results of Step 1, sort the polynomials into groups. Describe how you factor the polynomials in each group.

Step 3 Use your descriptions in Step 2 to factor the following.

a. $x^2 - y^2$ b. $x^2 - 2xy + y^2$ c. $x^2 + 2xy + y^2$

Figure 11. Connected use of CAS within the student textbook (Flanders et al., 2010, p. 746).

Although a CAS operates with symbolic forms, handheld calculators, which contain this feature, also possess a number of other functions. This aspect of the framework characterizes the nature of the results produced by the CAS-equipped handheld. For instance, the results can be numerical in nature when a spreadsheet, matrix, or home screen is used. The results may be of a graphical or geometrical nature when the dynamic geometry system on the handheld is used. Within the CAS functionality results may be formulaic as seen in Figure 7 above or non-formulaic. That is, the results involve a combination of numbers and variables as depicted in Figure 6. The variety of uses of the CAS-equipped handheld were also coded and placed into the following categories: number, matrix, spreadsheet, function, and graph/geometry.

Analyses of the text also revealed that although a CAS machine was used frequently, it wasn't always the primary tool used to solve a problem. In some instances it was and when this occurred it was coded as primary tool, while in other cases when it was used as just one step in the solution process it was coded as secondary. In other cases, the CAS wasn't used to solve a problem, but it provided a backdrop for students' problem solving capabilities. For instance, students might be asked to make sense of a result produced by a CAS for some other problem.

The nature of the how the CAS was used was also coded and placed into one of four different categories: equivalent expressions, equivalent equations, parameter experimentation, or pattern generation. As in the other aspects of the framework, these uses were developed from a combination of examining articles on technology as well as examinations of the textbook itself.

Reasoning and proof. Stylianides (2008) describes a framework that he used to examine reasoning and proof within a middle school reform-oriented mathematics curriculum developed in the United States. His framework involved three different components: mathematical, psychological, and pedagogical. This study involves several adaptations of his mathematical component. These adaptations included testing conjectures and the use of technology to assist students in identifying patterns, developing and testing conjectures, and fashioning proofs. A textbook question or imperative directed towards students was distinguished from a conjecture if it contained some aspect of uncertainty (e.g., could or may). In addition, technology was considered as helping students to develop a proof if it was used to complete a procedure that the students had demonstrated paper-and-pencil proficiency with involving formulaic forms.

Curricular change due to technology. Cole and Griffin (1980) describe the use of technology as an amplification of human capabilities. They give an example of a pencil as a form of technology that amplifies students' abilities to remember a long list of words. Pea (1985), on the other hand, describes the use of technology as a reorganizer that has the potential to reorganize how individuals think. He goes on to give examples of how an electronic spreadsheet fundamentally restructures the budgeting process by making planning and hypothesis testing the primary mental operations instead of the quantitative process of developing a budget. These ideas were used to consider the effect of CAS on the curriculum as well as thinking more specifically in terms of pedagogy, sequencing of mathematics materials, mathematical processes (e.g., problem solving), and mathematical content. The UCSMP *Advanced Algebra* (Flanders et al., 2010) makes a good candidate to examine curricular influences due to CAS, as the second edition of *Advanced Algebra* (Senk et al., 1996) does not include this technology.

A hard copy of the teacher's edition of the *Advanced Algebra* (Flanders et al., 2010) was examined page-by-page for indications of CAS use. The word CAS or screen shots of a CAS indicating that this technology had been used in the solution of a problem were used to locate CAS components that were subsequently analyzed using the framework described above. Graphing calculator use was also noted. If a question involved technology use that could be attributed to a graphing calculator it was coded as such, but if the screen shot or questions asked suggested CAS use it was coded as this technology. Components were located at the activity or student problem level. Each component was coded in the textbook using post it notes. Later the location, codes, and justifications were placed into electronic tables. These tables were saved as text files and were coded using HyperRESEARCH (ResearchWare, 2009). Frequency reports were developed using this software.

Results

An initial analysis of the textbook reveals that 63% of the technology occurrences involved CAS use with the remainder being graphing calculator use. The focus of this paper is on CAS use so the rest of the analyses will focus on this technology use in the curriculum. The majority of the CAS uses were active (62%) as opposed to passive. The CAS depicted in the textbook is generic. However, CAS in general, possess a variety of different capabilities as described earlier. Table 1 shows the frequencies within these different categories. The majority of the machine specific uses were in the area of function with matrices appearing second often. This finding exemplifies the recent goal of reorganizing a textbook around the big idea of functions described in the most recent set of standards from NCTM (2000).

Table 1

Machine Specific Capabilities

Capability	Frequency
Function	28
Matrix	22
Graph/Geometry	13
Spreadsheet	10
Number	10
Total	83

A summary of the different roles in which the CAS was used appears in Table 2. The majority of roles for the CAS were devoted to equivalent equations with expressions as a close second. Although functions were used frequently in terms of the machine specific capabilities described above, parameter experimentation was used quite infrequently to investigate the nature of different function families.

Table 2

CAS Roles

Role	Frequency
Equivalent Equations	81
Equivalent Expressions	41
Pattern Generation	14
Parameter Experimentation	4
Equivalent Inequalities	2
Total	142

The textbook authors used CAS as the primary means of solving problems (67% of occurrences) as demonstrated in the textbook or in problems directed towards students with 67% of occurrences. CAS was a secondary means of solving problems in 32% of the occurrences and in only 1% of the cases was CAS used as a backdrop in the problem solving process.

The nature of CAS results in the textbook are shown in Table 3. The vast majority of CAS uses were in the area of non-formulaic results with numeric results being the second most frequent category. Despite the CAS's proficiency with formulaic forms this appeared least frequently among the different categories.

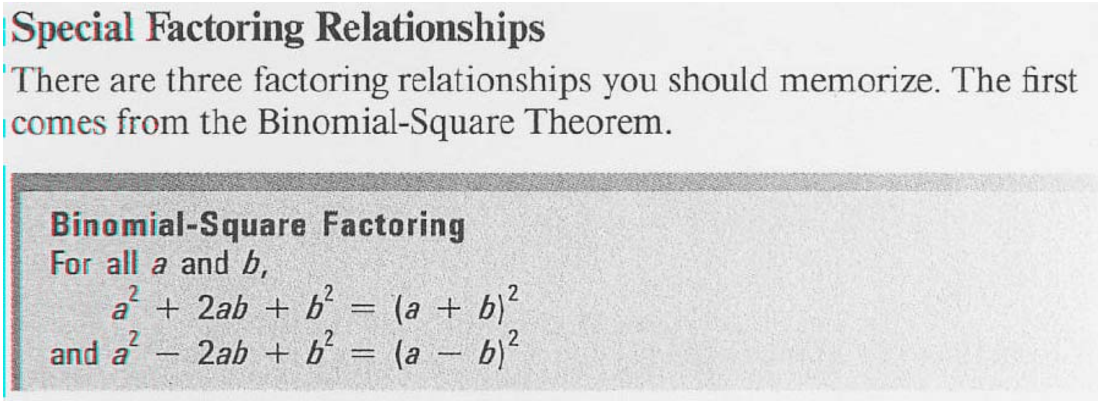
Table 3

Nature of CAS Results

Category	Frequency
Non-formulaic	135
Numeric	55
Graph/Geometry	19
Formulaic	16
Total	225

Out of a total of 225 CAS episodes, 69 involved some component of reasoning and proof or 31%. CAS was used most frequently to identify patterns and least frequently to test conjectures. This is somewhat surprising as it was thought that technology would be used least frequently in argument development. Another interesting finding is that it was hypothesized that the categories for conjecture development and conjecture testing would be roughly equivalent as students would test the conjectures that they developed, but students were asked to use a CAS roughly four times as often for developing conjectures as testing them. Consequently, there appeared to be a mismatch between the actions of conjecture development and conjecture testing.

The only area in which there was evidence to conclude a curricular change due to the use of CAS was within the pedagogical category. For instance, the second edition of *Advanced Algebra* (Senk et al., 1996) simply stated the Binomial Square Factoring theorem as seen in Figure 12.



Special Factoring Relationships

There are three factoring relationships you should memorize. The first comes from the Binomial-Square Theorem.

Binomial-Square Factoring
For all a and b,

$$a^2 + 2ab + b^2 = (a + b)^2$$

and $$a^2 - 2ab + b^2 = (a - b)^2$$

Figure 12. Formal presentation of the binomial-square factoring theorem (Senk et al., 1996, p. 687).

This theorem was “discovered” by students with the use of CAS in the activity depicted in Figure 9, taken from the third edition of *Advanced Algebra* (Flanders et al., 2010). There were no significant changes in terms of sequencing and mathematics content. Due to some of the uses of the CAS as seen in Figure 11 and in other places where students were asked to notice patterns they developed more proficiency identifying algebraic forms instead of simply applying algebraic procedures. The chapters and the order in which they appear within the second edition is nearly identical to that within the third edition, thus there were few instances of content or sequencing curricular changes due to the inclusion of CAS.

Discussion

This study presented a framework that could be used to examine the use of CAS within a variety of different curricula. One of the observations of these findings is that the percentages for some categories of CAS use as well as the use of technology to promote reasoning and proof certainly could have been higher. This is seen in the category of pattern generation. That is, there could have been a greater integration of CAS use within the curriculum. For instance, in the unit on logarithm properties students could have been asked to notice a pattern in the logarithm of different quotients with the aid of a CAS, develop and test conjectures, and eventually produce a proof for the logarithm of a quotient theorem. Instead, students were presented with the theorem in a highlighted box and asked to complete components of the proof in a student exercise.

Comparing the effects of CAS integration between the second and third editions illustrate that the textbook authors conceived of the technology as an amplifier of human cognitive capabilities instead of the more powerful reorganizer envisioned by Pea (1985). These findings are similar to those reported by Davis and Fonger (2010) who analyzed CAS instances within one unit from *Advanced Algebra* (Flanders et al., 2010) as well as two other reform-oriented textbooks. While the textbook authors took chances by integrating a technology into the curriculum that many teachers in the United States have been reluctant to use, this tool was harnessed to achieve traditional ends. The barriers described as the beginning of this paper may have played a role in the textbook authors' decisions around the use of this technology in the curriculum.

The results described in this paper could be used as a benchmark for analyzing other curricula that currently use CAS either in the United States or in other countries. This would permit comparisons between curricula organized for different purposes. An important question with regard to this framework is what are the ideal frequencies within each of the categories for a specific curriculum? Does this depend on the goals of the curriculum? If so, how? A future step in research in this area would be to examine the use of CAS within the enacted curriculum and compare those with the analyses conducted on the written curriculum.

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