## (TME-2010 - Conference; Malaga (Spain)

## Mathematics Lessons and Classroom examples, inspired by "Dresden Morning Post"

What is the connection between mathematics lessons and "Dresden Morning Post", an infamous newspaper in Saxony?
Students usually read this newspaper and so teachers can choice a topic or a problem which has any reference to mathematics. Starting out from the information of the "Morning Post" one should check the teaching curriculum and the teaching objectives. And so you can start to create an interesting problem for the lesson.

Why is it necessary to reform teaching of mathematics? We see tree reasons:
-„rigid" picture of mathematics

- availability of new media
-call for a new assignment culture
You see in this picture a typical side from a German mathematics book. You perhaps cannot recognize everything. This isn't so bad. Simply enjoy the Eroticism of these terms. Imagine a 14-year teenager now: He has so a "big interest" to experienced, what comes out at these terms and roots.

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    B. Bestimme aut 2 Dim
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13 a) }\frac{\sqrt{}{x}-\sqrt{y}{y}}{\frac{1/5}{5}
```

We think about a gradual change of the assignment culture in Saxony at present.
You should know: Saxony has central school leaving examinations and graphic calculators are obligatorily specified.

To chance the "picture of mathematics" we defined in our new curricula the following general goals of teaching mathematics:
-Advancement of problem solving competence
-Critical use of reason
-Advancement of the competence of using mathematical language appropriately.
-Advancement of the ability to visualise things
-Advancement of the competence of using basic mathematical objects appropriately
Graphic calculators are obligatorily specified in Saxony. The reasons for using CAS and graphic
calculators are the following didactic reasons:

- explorative learning - experimentation
- visualisation
- motivation
- use as a Calculator
- change of assignment culture
- cross-curricular teaching and learning


## I would like to illustrate it with some examples: Example 1: Football player

The goal of the example: visualizing, motivating
In October 2001 was a match between Germany and Finland. The result was 0:0 and the press was disappointes and enraged.
That was not particulary funny.
Especially the German player Oliver Bierhoff was in the centre of the criticism, because he did not hit the goal from a distance of 8 meters.
A student asked "Why 8 meters? Is it more terrible to miss the goal from a distance of 6 meters or 10 meters?


The forward goes along a imaginary line, parallel border of the field in the direction of the opposing goal. We estimate the distance from this line to the goal about 5 m . The breadth of the goal is $7,32 \mathrm{~m}$.

Look for the function: "distance of the forward from the demarcation of the field in line with the goal $\rightarrow$ angle of the shoot into goal".
short: distance $\rightarrow$ angle
Exist a maximum for this angle for the shoot into the goal?
What is the optimal distance for shooting?


We solve it with the TI-Nspire with geometry, spreadsheet and graphic and get: (Distance for shooting: $x$-axis, angle: $y$-axis)

$$
\alpha=\alpha_{1}-\alpha_{2}=\tan ^{-1}\left(\frac{12,32 m}{x m}\right)-\tan ^{-1}\left(\frac{5 m}{x m}\right)
$$




The optimal shooting distance is 8 metres, "Dresden Morning Post" was on the right track!!
The optimal angle is about $25^{\circ}$.
What a surprise!

## Example 2: Attention Toads

The goals of the example: open tasks, experimenting, visualizing
If I drive home with my car, I see daily a traffic sign on my way: Speed limitation on $30 \mathrm{~km} / \mathrm{h}$ and an additional sign on which a frog is shown.
After traffic order in Germany the additional sign indicates those traffic participants to whom the speed description applies.
If a bus is for example shown, then it applies to busses. If a motorcycle is shown, then it applies to motorcycles.
But in this case there is a frog on this sign!
If the police there had stopped me, I would have been very astonished.

However, the cause for the classroom-example was an article in the "Dresden morning post". The danger for the toads was described if they cross the street.
A toad needs up to 20 minutes to cross a road that is 7 m wide.


Dresden Morning Post

With students of a 10th class we wanted to discuss the danger for the toads and the use of the sign. We worked into groups with 4 students each. At first an idea was prepared in the class together:

Underlay e. g. 200m street with a decimeter raster.
How often (in seconds) does the scheme of bouncing change?


The first group calculated:

$$
\frac{70 \mathrm{dm}}{20 \mathrm{~min}}=\frac{1}{x \min } \Rightarrow x=0,285 \min =17,1 \mathrm{~s}
$$

The scheme of bouncing changes every 17 seconds.
How long does a car need to go 200m depending on speed?

$$
\mathrm{s} \text { in } \mathrm{s}=\frac{200 \mathrm{~m}}{\left(\frac{\mathrm{x} \frac{\mathrm{~km}}{\mathrm{~h}}}{3,6 \frac{\mathrm{~m}}{\mathrm{~s}}}\right)}
$$

At the picture you see the way in dependence of the speed:

$y(1)=200 /(x / 3.6)$ in steps of $10 \mathrm{~km} / \mathrm{h}$ :
How many of the „bouncing schemes" is the car coasting during this time?

Meter-reading of the intersection at the multiple of 17 says:


For example:
85 sec . ( 5 schemes) at $8,5 \mathrm{~km} / \mathrm{h}$
68 sec . ( 4 schemes) at $10,6 \mathrm{~km} / \mathrm{h}$ )
51 sec . (3 schemes) at $14,11 \mathrm{~km} / \mathrm{h}$ )
34 sec . ( 2 schemes) at $21,2 \mathrm{~km} / \mathrm{h}$
17 sec . ( 1 scheme) at $42,6 \mathrm{~km} / \mathrm{h}$ )


The result: „The more I slow down the more toads I hit."
„At 42,6 km/h and faster, it does not matter, there is no difference anyway."

An other group investigated the question: What influence does the reaction rate of the toad have?
Toads are able to see objects up to a range of 4 meters, to react on them within 0.5 sec . and to jump in case of emergency.

Distance of the car within 0.5 sec .:


This means: The covered distance of the car is 1.38 m at a speed of $10 \mathrm{~km} / \mathrm{h}$ and so on. Is this distance smaller than 4 meters, the toad is able to react.
solve ( $\mathrm{v} / 3.6 * 0.5=4, \mathrm{v}$ ):

$$
\mathrm{v}=28,8 \frac{\mathrm{~km}}{\mathrm{~h}} \approx 30 \frac{\mathrm{~km}}{\mathrm{~h}}
$$

Linking to the former model


The graph only exists from $30 \mathrm{~km} / \mathrm{h}$ up. There I coast appr. 1.4 „bouncing schemes" and therefore hit the highest possible number of toads."

An other group investigated the question: What influence does the stopping distance of the driver have? Stopping distance according to driving school rules:

| Speed | Stopping distance in m |
| :--- | :--- |
| 10 | 1 |
| 20 | 4 |
| 30 | 9 |
| 40 | 16 |
| 50 | 25 |

But there is the question in which distance is it the driver possible, to see a toad on the street.
Now the pupils called to the Dresden traffic police and get the following answer:

- You must not exclusively see the danger for the toads.
-Because of the slobber that is produced by driving over the toads the wheel grip is reduced to an extent that is comparable to aquaplaning. As a result you could lose control over the car. The signs are for your own safety.


## Example 3: Streetcar company

Goals: Visualizing, motivating, experimenting
An other Example from the "Dresden Morning Post":


## "The streetcar company in Dresden is merciless! institute 5000 court proceedings against fare doger!

In the message the Morning Post called a Passenger without ticket: fare dodger and postulate: "Every passenger should be checked average at least once in three month." Some facts:
Everyday 480000 passengers use the streetcar. With a honest passenger the company earn $0,40 €$, with a fare dodger $40 €$ fine (punishment).
The 456000 honest passengers pay every day $182400 €$, the 24000 fare dodger had to pay $960000 €$, if a streetcar-company guards would catch them all.

But:
If all 480000 passengers would pay honestly, the streetcar-company would only earned $480000 \cdot 0,40 €=$ 192 000€.

The Problem is:
The company would have to check the tickets exact so often, that the company get sufficient money and the fare dodger would be preserved.

Which number of inspectors is necessary for the biggest profit of the company?
We look at a period of time of three month, nearly 180 trips for a student. Assumptions:At the beginning there are $5 \%$ fare dodgersIf a fare dodger would be caught at least two times in this period, he will change into a honest passengerIf a honest passenger would never be controlled in this period, he will change into a black driver in the next time.

X is the number of checks in the period for any passenger. The distribution of X is binomial with $\mathrm{n}=180$ and unknown probability p. p is the probability, that a passenger would be controlled.

$\mathrm{y} 1=0.05 \cdot \operatorname{binomcdf}(180, x, 1)+0.95 \cdot \operatorname{binomcdf}(180, x, 0)$ und
$y 2=0.05 \cdot(1-\operatorname{binomcdf}(180, x, 1))+0.95 \cdot(1-\operatorname{binomcdf}(180, x, 0))$.
We examine the functions into dependence of the probability P .


Z : profit of the streetcar-company in the period for any passenger in dependence of the probability p

| $z_{i}$ | $0,40 €$ | $40,00 €$ | $0,00 €$ |
| :--- | :--- | :--- | :--- |
| $P\left(Z=z_{i}\right)$ | $y 2(p)$ | $p \cdot y 1(p)$ | $1-(p \cdot y 1(p))-y 2(p)$ |



For $\mathrm{p}=1,6 \%$ earn the company the highest profit.
Compare the model and the reality:

|  | model | reality |
| :--- | :--- | :--- |
| number of inspectors | 13 | 30 |
| number of checks <br> (600 per inspector <br> per day ) | 7800 | 18000 |
| check-probability p <br> (480 000 passengers) | 0,01625 | 0,0375 |
| average earn of the <br> company per passenger <br> (Z) | $0,415 €$ | $0,40 €$ |
| daily earn of the company <br> (480 000 passenger) | $199200 €$ | $192000 €$ |

result: If the streetcar-company would dismiss 17 inspectors, they would earn per day $7200 €$ more.

## Example 4: Traffic Jam

Goals: Visualizing, motivating, experimenting


Questions of the "Dresden Morning Post":
Why is everyone dawdling?
Couldn't feel faster if wasn't the truck in the street?
distance $=$ length of the car $a+$ safety distance
Distance $=$ length of the car $+a+\frac{1}{2} \cdot\left(\frac{v}{10}\right)^{2}$
(This is the formula for the safety used on drivers school.)
We examine the connection:

| Speed (km/h) | necessary distance of the car <br> in m | Place is up on 200 m of street |
| :---: | :---: | :---: |
| 0 | 5 | 40.0 |
| 10 | 5.5 | 36.3 |
| 20 | 7.0 | 28.5 |
| 30 | 9.5 | 21.0 |
| 50 | 17.5 | 11.4 |
| 70 | 29.5 | 6.8 |
| 100 | 55.0 | 3.6 |
| 130 | 89.5 | 2.2 |
| 180 | 167.0 | 1.2 |

In which time passes the car the necessary distance of it's own?

$$
t(v, L)=\frac{L}{v}=\frac{3,6 \cdot\left(a+\frac{1}{2} \cdot\left(\frac{v}{10}\right)^{2}\right)}{v}
$$



We examine it with the TI-Npire and find, the optimal speed 5 m -cars is nearly $30 \mathrm{~km} / \mathrm{h}$. If you observe trucks with a lengths of 20 m , the optimal speed is nearly $60 \mathrm{~km} / \mathrm{h}$.

## Example 5: Bypass Street

Goals: discovering, experimenting•There are two districts in the city of Meißen, connected by two bridges. The car drivers have to possible routes, about a and c or about b and d.
Now a new freeway was built.
But we can observe more traffic jam in Meißen! (The Dresden Morning Post report!)
The old Situation:
In the morning 1000 cars moved from A to B every day
To oppurtunities: route a and b or route c and d


For the roads c and b you need ever 15 minutes. But on the bridges is every morning traffic jam. From experience one knows: one needs $\mathrm{x} / 100$ minutes at x cars (at more than 100 cars).

If 1000 drivers would decide in favor of route $a$ and $b$, they would need 15 minutes $+1000 / 100$ minutes $=$ 25 minutes.
How should the drivers spread out?
x: Route a and b
y : Route c and d
Equation 1: $x+y=1000$
Equation 2: $\frac{x}{100}+15=\frac{y}{100}+15$

The solution is easy: $x=y=500$
No surprise, the half of the drivers must choose one route each.
Necessary time: $\frac{500}{100}+15=20$ minutes
Balance adapts between a and b drivers and c and d drivers.

## Now the New situation:

A freeway was built with only one entry in the north and only one exit in the south.
3 Bridges: a, d and the freeway e.
For the freeway e you need only 7,5 minutes, for the other ways nothing has changed.


3 kinds of drivers:
X : route a and b
Y: route c and d
Z: route aàeàdWhat's the optimal distribution of the vehicles?
( 1 ): $x+y+z=1000$
(II): $\frac{x+z}{100}+15=\frac{y+z}{100}+15$ Solution with TI-Nspire:
(III): $\frac{x+z}{100}+15=\frac{x+z}{100}+\frac{y+z}{100}+7,5$
$X=250, y=250, z=500$
Running-time of the 3 groups:
$\frac{x+z}{100}+15=\frac{y+z}{100}+15=\frac{x+z}{100}+\frac{y+z}{100}+7,5=22,5$ min utes Now you need 2,5 minutes more than before the opening of the freeway.
Cause: Bottlenecks to the beginning and the end of the way, there is no Simplification.
further examples of bypasses planned badly
Stuttgart center (Sclossplatz) $\rightarrow$ more jam
New York, inhibition of the 42nd street $1990 \rightarrow$ less jams
Rügendamm in Germany $\rightarrow$ more jam
Literature:

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