Physics Through GeoGebra Window

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ABSTRACT

We start seeing physics (Nature, the surrounding environment) with naked eye then we would resort to use any available device to see better and to get more insight into 'physics'. The commonly used devices are paper and pencil; one can draw figures and curves and do some calculations followed by some conclusions. But since holding a physical handle is easier than confining ourselves to the paper and pencil, we do experiments to improve our understanding. Continuing this method we may design experiments that need more expenditure of time and effort. Hence we resort to the virtual world in order to see the real world. In this paper I present some virtual experiments by GeoGebra that include the electromotive force induced in a loop while entering a uniform magnetic field which first done by MATLAB and locating the focus of spherical mirrors that first done by graphing calculator CASIO CFX-9850G then continued by GeoGebra due to the efficiencies and the simplicity in using this software. Also image formation in spherical mirrors, the relation between refractive index and the deviation angle in the prism and the condition for light emergence from the prism, inclined plane that acts as a simple brachistron, a euro that rolling around stationary euro, and the speed of a slipping ladder are investigated.

These experiments are more flexible than their real counterparts. For example the dimension of the spherical mirror can easily be changed to see its effect on the focus position or to see when the spherical mirror equation hold. In addition to the ease of changing the refractive index and the prism angle, the curve of relation between deviation angle with the incident angle traced simply by changing the incident angle and the user can check the arrangement that satisfies the mathematical relation that ties the two angles to the refractive index. As to the induced electromotive force it visualizes the moving loop and the electric wave which inspires a relation between the wave shape and the shape of the loop – I investigated such relation for circular loop mathematically, and for trapezoidal and triangular loops only

experimentally and due to their characteristics, I called these virtual tools virtual signal generator. The rectangular loop is clearly stated in the textbooks as an example or an exercise.

Keywords

Physics, virtual experiments, GeoGebra, graphing calculator, Dynamic geometry, MATLAB.

Introduction

In August of 2006 Dr. Trigo [2] had an interesting presentation of examples of usage of Cabri in mathematics teaching and he explained that the educational system of Mexico was working on implementing the dynamic geometry in mathematics curriculum. After that session I searched for Cabri but I did not find it so I forgot it completely. Three years later, in July 2009, professor Zsolt Lavicza gave me the web address of GeoGebra. It took me less than one month to stick to GeoGebra – the simple and powerful software. GeoGebra Wiki and forum provide many works of many students and teachers that mainly deal with mathematics teaching and learning but physics problems and exercises are comparatively rare and sometimes not presented in a direct manner, for example, image formation in spherical mirrors was carried out in accordance with the spherical mirror equation and the focus fixed exactly at the middle point of the center and the vertex without any consideration about the mirror dimension that can accept such arrangement and equation.

The following examples give a clear reason to open the Geogebra windows towards the physics teachers, and the students that may do some physics, for the sake of more fresh teaching and learning methods.

The Sliding Ladder

The end of a ladder that leans against a wall moves uniformly downward. Will the other end move uniformly too? The familiar solution to this problem is $v_x = \frac{dx}{dy}.\frac{dy}{dt}$ and $x = \sqrt{l^2 - y^2}$ then $v_x = \frac{-y}{\sqrt{l^2 - y^2}}.v_y$ but why does v_x get smaller and smaller as y tends to zero? The answer becomes clear if we watch the ladder from the instantaneous center of rotation. As the user moves the upper end of the ladder downward, the instantaneous center of rotation goes nearer to the end of the ladder therefore the ladder almost rotate around its lower end and the vector of velocity –

seen from the instantaneous center of rotation – almost becomes parallel to the wall (y-axis), see figure 1. An slider introduced to make the motion more smooth and to hold the ladder from penetrating the ground.

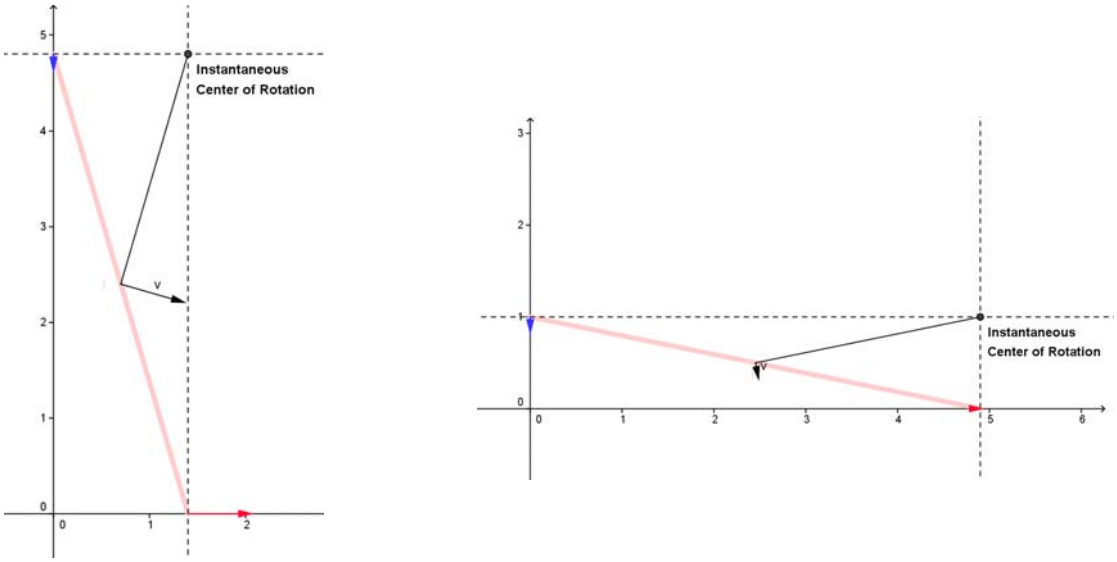
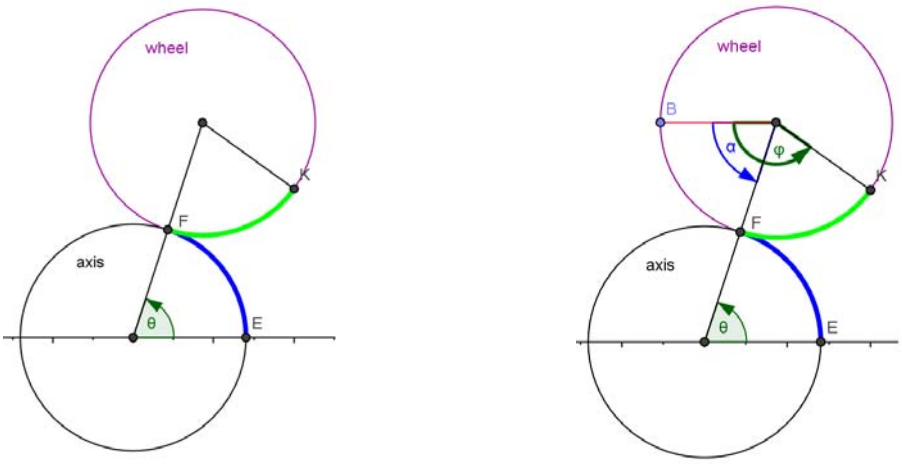


Figure 1. Two snapshots of the sliding ladder.

The Rolling Euro

How many revolutions will a euro make when rolled completely around a stationary euro? Or does the earth rotate 360° in a day?

The common sense would be happy with one revolution for the rolling euro or 360° per a day for the rotating earth. This is true if we watch the rolling objects from the instantaneous center of rotation. As figure 2 (a) shows the lengths of arcs FK and FE remain the same at any instant and if the radiuses of the two circles are equal then the corresponding angles also are equal which indicates that only one revolution would be done.



(a) – seen from instantaneous center of rotation. (b) – seen from a stationary point as the earth seen from a distant star.

Figure 2. A circle is rolling around a stationary circle with the same radius.

Now, let's look at the rolling objects from another point. The motion starts from x-axis so the angular displacement of the radius that coincides with this axis at t = 0 to be investigated. Figure 2 (b) shows a picture of this angular displacement which is greater than the angular displacement seen from the instantaneous center of rotation by α. Check α variations during the motion. If you satisfied with a euro rolling around another one, save the two and continue working with GeoGebra simulation; change slider n to 0.5 (use arrow keys for fine adjustment) and guess the number of revolutions that would make by the rolling circle. Instead of continuing reading let's break for trying different values of n, and to switch from guessing to formulating. Ok, as you noticed the length of arc FE in figure 2 (b) is $R\theta$ and for FK is $nR\varphi$ - $nR\theta$ and equating these quantities leads to:

$$\varphi = \left(1 + \frac{1}{n}\right)\theta \qquad (1)$$

According to equation (1) we would get one revolution if n tends to infinity. But what is the infinity in physics? Compared to the electrons-nucleus distance a millimeter is infinity. In some circuits a fraction of a second is infinity. Choose a very small value for R and a large value for n and find a reasonable value for infinity – a large gear that rolls a turn around a thin axis, rotates approximately one turn around its center.

Simple Brachistochrone

Two inclined planes coincide with the chords of the same circle of radius R. A small body slides down each of them without friction and without an initial velocity. For which of the planes is the time of sliding greater? [4]

A slider controls the gravity acceleration. Move point C, the release point, to change the inclination of b and watch the elapsed time that visualized as dynamic text. Repeat this experiment with point E. For the moving point C there are infinite number of isochronous paths – with different lengths and equal elapsed time. There is no maximum or minimum of time so each path may be considered as a simple brachistochrone that consisted of a chord of circle instead of being a piece of cycloid.

This is a good starting point to encourage students to think about a path that would be isochronous for any starting point on it – the brachistochrone.

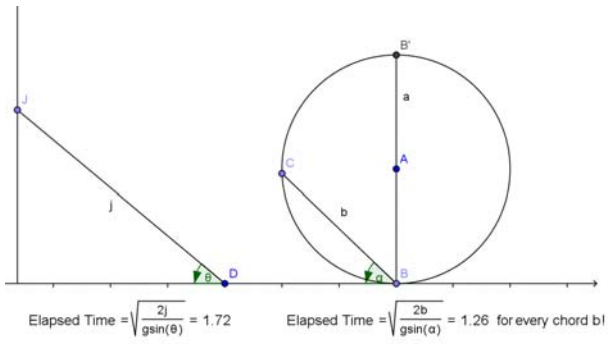
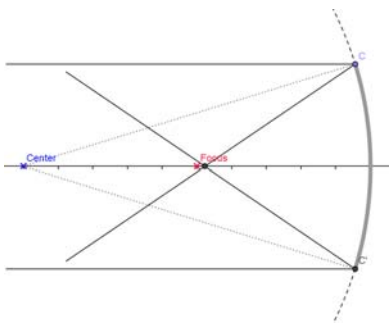


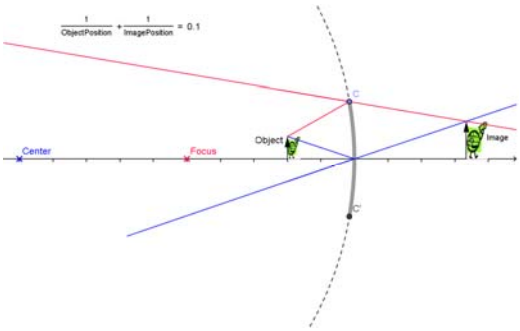
Figure 3. A Simple Brachistochrone compared to an inclined plane.

Spherical Mirrors

Spherical mirrors are a good subject of practicing geometry and applying good approximations to the real word. The golden rule in constructing a simulation for these mirrors is to use only the law of reflection. Such constructions are depicted in figures 4 (a) and (b). Figure 4 (a) shows light rays parallel to axis of the mirror and their reflections that converge on the axis. The convergence point, by definition, is the focus and it reaches the midpoint of the center and vertex only when the length of mirror arc tends to zero [1]. Don't stick to intangible thinking. This zero has the same story of the infinity in physics. Why the zero can not be more than one while the infinity can be less than one? Tick the 'Check Mirror Dimension' box and move the upper end of the mirror arc and check the live text to see when does f equal r/2. The mirror dimension that satisfies this equality changes with varying accuracy (chosen from the menu options / rounding).



(a) Setting a proper dimension for the mirror.



(b) Image formation in the mirror.

Figure 4. A spherical mirror.

After setting a proper dimension for the mirror, conduct some experiments simply by changing the object position. Check the arrangements that satisfy the spherical mirror equation and the equation of linear (lateral) magnification. Check questions like: Why do the equations of spherical mirror and linear magnification work properly for some values of object position and fail for some others values? Why the focal length equals one-half the radius of curvature? Why do we use paraxial rays in studying image formation in spherical mirrors? Why the spherical mirror is a very small portion of a sphere? I started constructing this virtual experiment for concave mirror but it works properly for convex mirror, why?

Prism

Prism includes some parameters tied together with simple but precise computations that when seen from the window of dynamic geometry GeoGebra will reveal beauties similar to the light spectrum formed by the prism. Figure 5 shows a monochromatic ray traced through a prism by refraction laws.

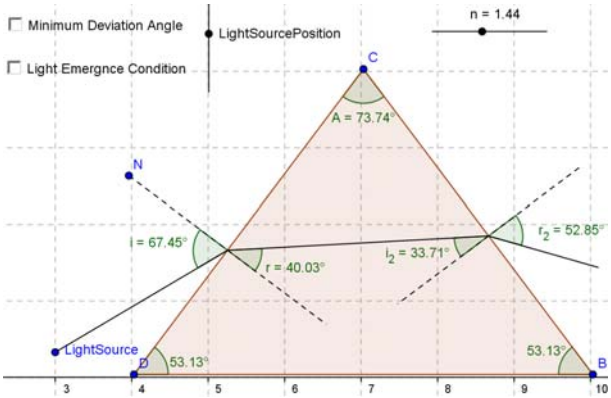


Figure 5. The trace of a monochromatic ray in a prism.

Here, the condition of light emergence from the prism and the condition for which the equation

$$n = \frac{\sin\left(\frac{D_m + A}{2}\right)}{\sin\left(\frac{A}{2}\right)} \qquad (2)$$

holds are investigated. Let's check the emergence condition first because there would be no deviation angle when light can not emerge from the prism and the equation (2) would make no sense. Only when $i_1 < i_c$ the light will emerge. i_c is the critical angle

and connected to the refractive index by $\sin i_c = \frac{1}{n}$ but it is evident from the geometry of figure 5 that A = r + i_1 and the light emergence condition becomes

$$A < r + i_c \qquad (3)$$

Sebenne and Balkanski [5] used refraction laws and considered the maximum value of the trigonometric function sine to obtain $A < 2i_c$. Choose light emergence condition and change the light source position to see that such condition always works but with a large blank interval. To skip the blank interval rewrite the inequality (3), using laws of refraction, as following

$$A < \arcsin\left(\frac{\sin i}{n}\right) + \arcsin\left(\frac{1}{n}\right) \qquad (4)$$

A point plotted on x-y plane with incident angle *i* as its x-coordinate and the right side of the above inequality as its y-coordinate. This point that its coordinates attached to it by dynamic text, traces a curve in $0 \le x \le \pi / 2$ that the acceptable values of angle *A* lie beneath it. Note that the condition of light emergence indicated in its y-coordinate. Condition of $A < 2i_c$ also shown as live text at the upper left corner. Compare the two conditions and see how the computer assisted teaching shortens the route of mathematical reasoning and eliminates the blank interval. Now, return to equation (2) which in some texts like Halliday's Physics [3] derived by assuming i = r_1 and in Síbenne's and Balkanski's Physics a detailed derivation is presented [5]. Figure 5 shows that $D = i + r_1 - A$. Tracing the light ray in reversed route – expressing r_1 in term of i – leads to:

$$D = i + \arcsin\left(n \sin\left(A - \arcsin\left(\frac{\sin i}{n}\right)\right)\right) - A \qquad (5)$$

Equation (5) treated as inequality (4). Choose 'Minimum Deviation Angle' box, vary the angle *i* and keep an eye on the dynamic text that shows equation (2). If you get the same number determined by slider n (you may need a longer format of number, controlled from option menu) then check the light rays; do you see any symmetry? Move point B to change the prism shape, and repeat the experiment; what about the symmetry? What about the position of the moving point that traces the curve? Examine the deviation angle while you change the angle i .

Virtual Signal Generator

A rectangular loop enters a uniform magnetic field **B** with a constant velocity **v**. Plot the induced emf [see example 2, pp. 580-581 of ref. 3]. According to Faraday's law of induction and the uniformity of **B**, the main mathematical idea in solving this problem is time derivative of an area of the loop that entered the magnetic field. To do so, it is sufficient to pass an area say a rectangle across a line $x = x_0$, see figure (6).

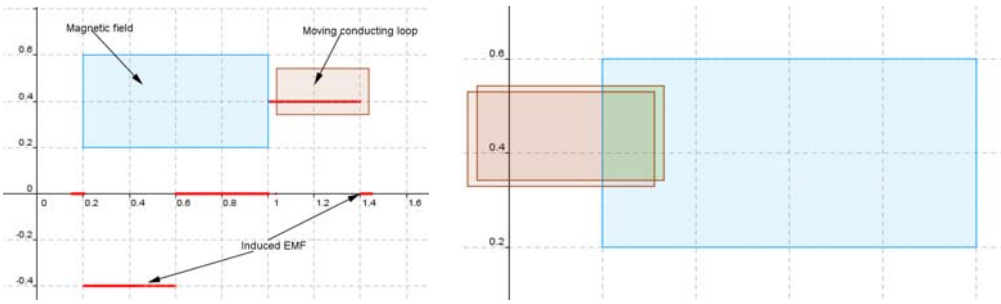


Figure 6. A conducting rectangular loop enters a uniform magnetic field.

The previous positions of the rectangle which is needed for calculating the differences are traced by another exactly similar area that lagged the first by a time difference Δt. In my experiments $\Delta t = 0.01s$ and the goal is to investigate the wave shape of induced emf so the values of the graphs and magnetic field **B** are not given in real scales. However one can calculate the real values, assuming B is 0.1 of values of slider *Mfield*, and attach them to the tracing points as live text – as I did with the prism. The wave shape in these experiments suggests good likeness of corresponding loop so this method can be used with any shape of loop to produce the desired wave – in theory, we have a signal generator that virtually can generate any form of waves.

Conclusion

Virtual experiments done by dynamic geometry can extend the physics laboratory to some limits that are not reachable in real world. Especially when repeating an experiment with different arrangements is needed for many times; changing the shape of a prism, varying the radius of a wheel or gear, looking at a sliding ladder from a point that shows its rotational movement, changing the radius and dimension of spherical mirror, and varying the index of refraction are good examples of such experiments.

To interpret the likeness in induced emf consider an area *A* produced with $y = f(x)$, a ≤ x ≤ b , and passing across a line $x = x_0$ then the time derivation of the passed area becomes

$$\frac{dA}{dt} = \frac{d}{dt}\int_{x_0}^{x(t)} f(x)dx = \frac{dx(t)}{dt}\frac{d}{dx(t)}\int_{x_0}^{x(t)} f(x)dx \qquad (6)$$

so

$$\frac{dA}{dt} = v \cdot f(x(t)) \qquad (7)$$

Thus for constant speed of a loop the rate of change of area with time follows a curve with some likeness of the curve of *f(x)*. It is clear that we can not represent a loop by a function but we can divide it into some pieces each produced by a function. For example a circular loop with radius *r* and center *(a,b)* can be created by

$$f_1(x) = b + \sqrt{r^2 - (x-a)^2}$$
$$f_2(x) = b - \sqrt{r^2 - (x-a)^2}$$

And according to equations (6) and (7)

$$\frac{dA}{dt} = v \cdot (f_1(x) - f_2(x)) = 2v \cdot \sqrt{r^2 - (x-a)^2}$$

As always, the math leads the technology by showing the right way that the technology must follow but how to deal with a loop like figure (7) (a)?

(a) (b)

Figure 7. A loop with a complicated shape.

Here, the integrand of equation (6) is unknown and it should be very complicated. Hence the technology would be the last refuge – one good turn deserves another. Mathematical rules guide the technology and the technology carries the mathematical loads. GeoGebra does not give the opportunity for creating such shapes. However, this is a good opportunity to ask the GeoGebra developers to add some

facility to enable us to draw shapes just as we do in Microsoft Paint and to know the area of something like the dyed zone in figure 7 (b) which enclosed between a part of the drawn shape and a vertical line.

A word of appreciation

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