The Positive Aspects of Modeling Process in Teaching Mathematics

Natalija Budinski

Primary and grammar school ”Petro Kuzmjak”
Ruski Krstur
Serbia

nbudinski@yahoo.com

TIME 2010, Malaga, Spain

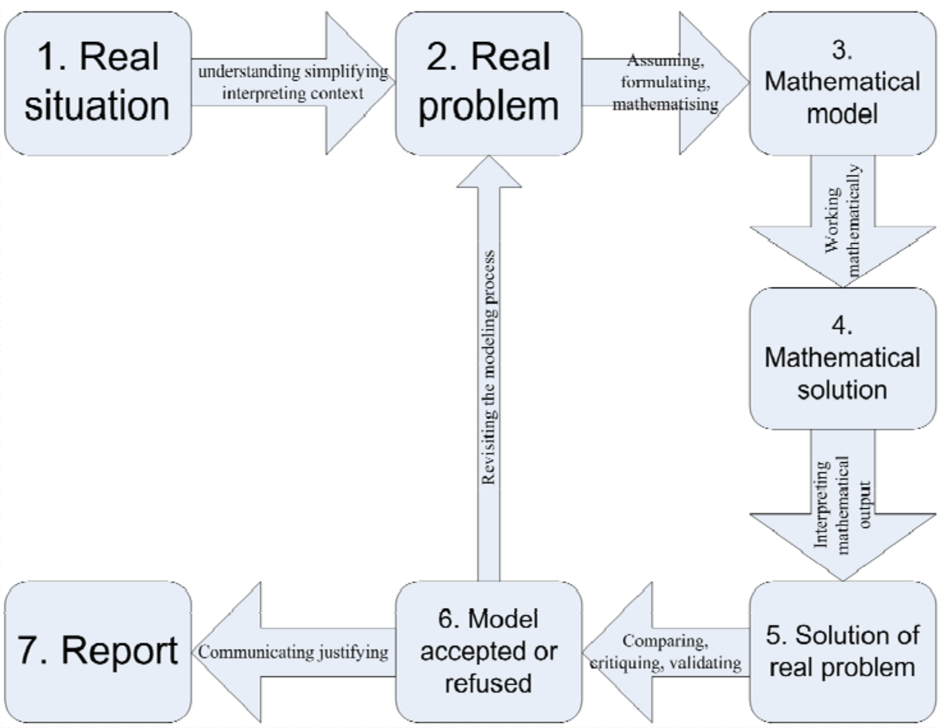
ABSTRACT

- This presentation proposes modeling based learning as a tool for learning and teaching mathematics. The example of modeling real world problems related to exponential functions and differential equations is described.
- The modeling process in mathematical education tends to follow the didactical cycle of activities and reach a desirable level of accomplishments of students’ activities and competencies.

Modeling in The Teaching Process

- Studying a mathematical model of a real-world situation can provide students with insights that are hidden during a nonmathematical study of the same situation.
- We use a diagrammatic representation (see Figure 1), which encompasses both the task orientation, and the need to capture what is going on in the minds of individuals as they work collaboratively on modelling problems.

Figure 1



“What are the objectives of modeling?”

Goals of modeling are

- prediction
- design
- the testing of possibilities in order to make a decision
- the development of a deeper understanding of a phenomenon

Big ideas could be presented in meaningful context, where modeling serves as a tool to bridge the different levels of mathematical understanding and, meanwhile, control the problem complexity.

Modeling the Differential Equation

- Two groups took part in the research during 2009/2010 school/ academic year
- Students from Grammar School ”Petro Kuzmjak”, Ruski Krstur, Serbia
- 19 students of the high school final year (18 years old) participated in the research. They were introduced with the basics of Calculus, as the basics of the Calculus are part of the National Curriculum for the final high school year
- Chemistry students from University of Novi Sad
- 16 first year chemistry students participated in the research. They were introduced with the Calculus during the first semester and also during second semester when they were having Analysis 2 Course.
- Both groups were not familiar with the differential equations

- We decided to initiate modeling activities to introduce modeling to our students in a belief that it could contribute toward better understanding of learning and teaching mathematics.
- The goal of this modeling process was to acquaint students with basics of differential equations and to make them aware of ODE in a profound way.
- In addition, the use of educational software was a very important part of the project.

Modeling population growth REAL SITUATION

- A classic example of modeling with first order ordinary differential equations is the population growth model. Similarly, exponential functions are frequently presented as models of population growth.
- To make differential equations close to the reality we took the population of Ruski Krstur as Grammar school is placed the Ruski Krstur.
- Students` task was to estimate population of Ruski Krstur trough modeling process.

The first step in the modeling process-REAL PROBLEM

YEARS	POPULATION OF Ruski Krstur
1948	5874
1953	6115
1961	5873
1971	5960
1981	5826
1991	5636
2002	5213

(sr.wikipedia.org/sr/Руски_Крстур)

MATHEMATICAL MODEL

Theoretical framework for modeling the population:

- Definition: **Derivate of the function**

The derivative is the value of the difference quotient as the secant lines approach the tangent line. Formally, the ***derivative*** *of the function ƒ at a is the limit*

$$f`(a)=\lim_{h\to 0}\frac{f(a+h)-f(a)}{h}$$

of the difference quotient as h approaches zero, if this limit exists.

If the limit exists, then ƒ is differentiable at a.

- Let us note with N the population number in the time that we observe and with k constant rate
- We observe the current population because further growth of the population depends on the initial conditions
- The following expression can be obtained:

$$N(t+\Delta t)-N(t)=kN(t)\Delta t$$

$$N(10+2)-N(10)=kN(10)\times 2$$

MATHEMATICAL SOLUTION

- Mathematical solution includes algebraic expressions which are simplified and factored, equations, which are solved and graphed.
- The mathematical solution of the students was following:

$$\frac{dN}{dt} \sim N$$

$$\frac{dN}{N} = kdt$$

$$\int \frac{dN}{N} = \int kdt$$

$$\ln N = -kt$$

$$\ln N = -kt + C$$

$$\ln N - C = kt$$

$$\ln N - \ln N(0) = kt$$

$$N(t) = N(0)e^{-kt}$$

SOLUTION OF THE REAL PROBLEM

- If we look at the equation that was obtained by modeling process there are some questions that could be discussed.
- We chose to discuss the problem of population deminishing.
- **Question:** *In which time the population of Ruski Krstur would dimidiate?*
- **Answer:** In order to give the answer students needed to calculate the following:

$$t = 0, N(0) = 5636$$

$$t = 11, N(11) = 5213$$

$$kt = \ln\frac{N(0)}{N(11)}$$

$$11k = \ln\frac{5636}{5213}$$

$$11k = 0.07819088$$

$$k = 0.007092644$$

$$kt = \ln\frac{N(0)}{N(t)}$$

$$t = \frac{1}{k}\ln\frac{N(0)}{N(t)}$$

$$t = \frac{1}{k}\ln\frac{1}{\frac{1}{2}}$$

$$t = \frac{1}{k}\ln 2$$

$$t = \frac{1}{0.007092644}\ln 2$$

$$t = 97.7\,god$$

Model accepted or refused

- Students who successfully completed the modeling process and came up with a good solution presented their work. At this stage, the strengths and weaknesses of the model were discussed.
- Questions and concerns were prepared by students who were not successful in the modeling process of this particular problem.
- Therefore, this was a constructive way to verify solutions and point out the weak spots of the model

Students` remarks

- Births are generally proportional to the size of a population. However, human populations rarely display such simple behavior for long periods of time.

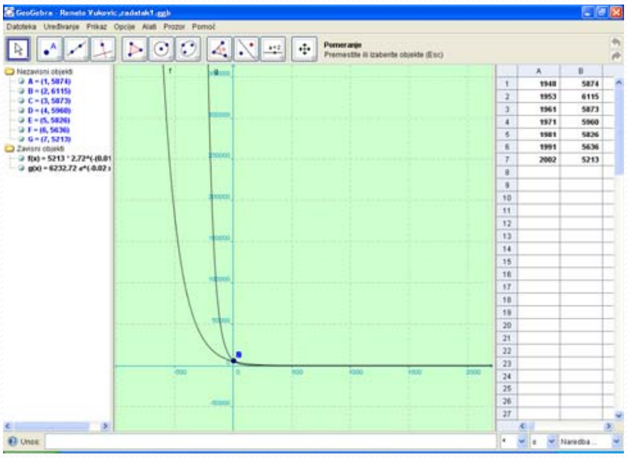
- Further cycles of modeling might incorporate other variables, such as changes in health and nutrition, immigration, wars, or the age distribution of a population.

Geogebra in the modeling process

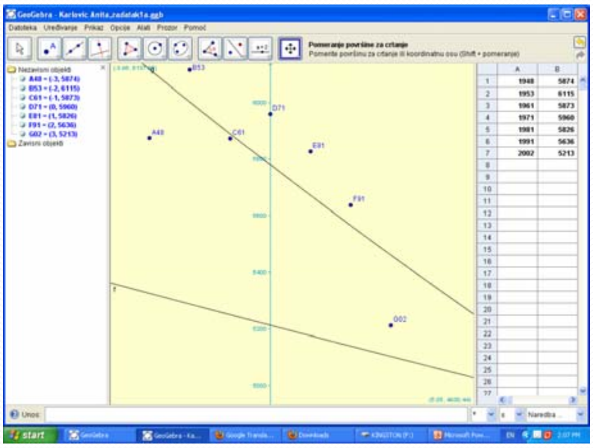
- GeoGebra affords a variety of representational resources and computational utilities that have the potential to engage students in insightful investigations of fundamental ideas of mathematics.
- GeoGebra provides an accessible platform to model and further simulate a variety of mathematical ideas, allowing for both expressive and exploratory ways of model-based learning .

Students` solution in GeoGebra

Renato`s solution



Anita`s solution



Concluson

- Students benefits from the modeling based learning.
- ICT in education need to be meaningfully integrated with modeling to enhance relevance of education.
- This teaching method tolerate different kinds of activities such as research every day situation, using computers and organizing the learning process close fitting to contemporary students and their field of interests.
- It can be said that modeling based learning is out of the box, but that is its great advantage over the traditional teaching

References

1. Mathematical Modeling: Teaching the Open-ended Application of Mathematics, Joshua Paul Abrams, www.meaningfulmath.org
2. Geogebra in mathematics teacher education: the case of quadratic relations, Lingguo Bu and Erhan Selcuk Haciomeroglu, MSOR Connections Vol 10 No 1. February 2010 – April 2010
3. Boaler, J., Mathematical Modelling and New Theories of Learning (2001), Teaching Mathematics and its Applications, Vol. 20, Issue 3,p. 121-128.
4. Doerr, H., English, L., A modelling perspective on students' mathematical reasoning about data, (2003) Journal for Research in Mathematics Education, 34(2), 110-136.
5. Galbraith, P., Stillman, G., Brown, J., Edwards, I., Facilitating middle secondary modelling competencies. (2007), In C. Haines, P. Galbraith, W. Blum, S. Khan (Eds.), Mathematical modelling (ICTMA12): Edu., eng. economics, Chichester, UK: Horwood, 130-140.
6. Henn, H.-W., Kaiser, G., Mathematical Modeling in school-Examples and Eperiences, (2005), In Mathematikunterricht in spannungsfeld von Evolution und Evaluation, Fesband fur Werner Blum, Hildesheim Franzbecker, 99-108 p.
7. Hohenwarter, M., Hohenwarter, J., Kreis, Y., Lavicza Zs., Teaching and calculus with free dynamic software Geogebra, *(2008), 11th International Congress on Mathematical Education, Mexico.*
8. Kaiser, G., Schwarz, B. Mathematical modelling as bridge between school and university, *(2006), ZDM, 38 (2), p. 196-208.*
9. Lamon, S. J., Parker, W. A., Houston, S. K. (Eds.), Mathematical modelling: A way of life, Chichester, *(2003), UK: Horwood*
10. Mason, J., Modelling modelling: Where is the centre of gravity of-for-when teaching modelling*? (2001), In J.Matos, W. Blum, K. Houston, S. Carreira (Eds.), Modelling and mathematics education, Chichester, UK: Horwood.*
11. Stillman, G., Brown, J., Challenges in formulating an extended modelling task at Year 9, *(2007), In H. Reeves, K. Milton, & T. Spencer (Eds.), Proc. 21. Conf. Austr. Assoc. Math. Teachers. Adelaide: AAMT .*
12. Stillman, G., Galbraith, P., Towards constructing a measure of the complexity of applications tasks. *(2003), S.J. Lamon, W. A. Parker, & S. K. Houston (Eds.), Mathematical modelling: A way of life (pp. 317-327). Chichester, UK: Horwood.*

- Thank you for your attention