

# Teaching and Assessing Polygons Using Technology

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Using dynamic geometry software to teach geometry can transform the way we approach teaching and learning. Meaningful technology based tasks allow students to discover mathematics on their own and construct their own understanding. Furthermore, they require active engagement of students and encourage higher level thinking. They challenge students to think about mathematical ideas in a new light.

In the following article we will show how technology helped us create an engaging learning environment for a unit on Polygons as well as share strategies that have helped us implement technology effectively. Through various age appropriate activities students explore polygons and their properties, make conjectures, test them, reason about geometric ideas as well as demonstrate understanding and ability to apply their knowledge of polygons.

For every lesson we prepare a „*Sketchpad notebook*”. This „notebook” contains basic theory, a few solved examples (step by step), as well as problems for individual exploration/learning. Class usually begins with a discussion and a presentation of examples followed by individual work by students. At that time, we introduce new terminology and concepts students are not able to discover on their own. Then students work independently or in pairs on the assigned tasks (see figure 1 and figure 2).

Fig. 1 Students use geometry software to explore the rules for the number of diagonals from one vertex of a polygon and the sum of diagonals in a polygon

Question 1: Draw the quadrilateral  $ABCD$  and all its diagonals. How many are there? Name them.

Question 2: Draw the convex pentagon  $ABCDE$  and all its diagonals. Name them. How many diagonals start (end) at point  $A$ ? How many diagonals does a pentagon have?

Question 3: Draw the convex hexagon  $ABCDEF$  and all its diagonals. Name them. How many diagonals start (end) at point  $C$ ? How many diagonals does a hexagon have?

Question 4: Draw the convex heptagon  $ABCDEFGH$  and all its diagonals. Name them. How many diagonals start (end) at point  $D$ ? How many diagonals does a heptagon have?

Question 5: What is the number of diagonals from one vertex of a polygon?

Question 6: What is the total number of diagonals in a polygon?

Fig. 2 Students use geometry software to explore central angles of a polygon

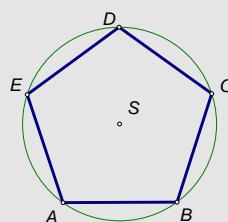
Problem 1: Construct equilateral triangle  $ABC$  and its inscribed and circumscribed circles. (The center of the circumscribed circle (circumcenter) is the point of intersection of the perpendicular bisectors of the sides. The center of the inscribed circle (incenter) is the point of intersection of three internal angle bisectors of a triangle.)

Problem 2: Measure the lengths of sides  $AS$ ,  $BS$  i  $CS$  and the measures of angles  $ASB$ ,  $BSC$  i  $CSA$ . What do you notice?

Problem 3: Construct square  $ABCD$ . Inscribe it and circumscribe it. (The centers of both circles are at the same point. Label it  $S$ .)

Problem 4: Measure the lengths of sides  $AS$ ,  $BS$ ,  $CS$  and  $DS$  and the measures of angles  $ASB$ ,  $BSC$ ,  $CSD$  and  $DSA$ . What do you notice?

Problem 5: Regular pentagon  $ABCDE$  is given. Point  $S$  is a center of its inscribed and circumscribed circle. Measure the lengths of the sides  $AS$ ,  $BS$ ,  $CS$ ,  $DS$  i  $ES$  and measures of the angles  $ASB$ ,  $BSC$ ,  $CSD$ ,  $DSE$  i  $ESA$ . What do you notice?



Problem 6. Complete the table.

$n$	5	6	9	20				
$\alpha_n$					$30^\circ$	$18^\circ$	$24^\circ$	$36^\circ$

Problem 7. Inscribe a square in the given circle.

Problem 8. In the given circle inscribe a regular hexagon.

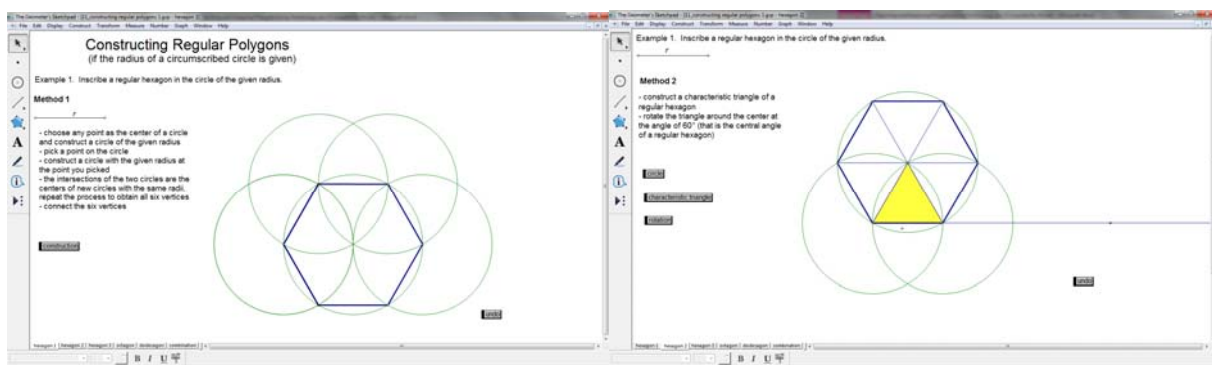
This is an excellent time for the teacher to observe students, the way they approach the problems, the strategies they are using to solve them and the conjectures they make. Listening to student interactions while working on problems offers valuable insight into student understanding. The role of the teacher during these explorations is to provide students with frequent feedback and guidance as well as intervene and clear up any misconceptions, if necessary (see figure 3).



Fig. 3 A picture of students exploring properties of polygons

At the end of class, students save their work on the server. If they did not have enough time to complete their work, they can continue working on it at home or after school. Once they have completed their assignment, they email it to their teacher. The teacher assesses student work and returns it with his/he comments and explanations on how to improve it, if necessary.

The next class, the teacher can show all the solutions and comment on them with students. In case students have difficulties solving the problems, the teacher can prepare a step-by-step presentation to show how the problems were supposed to be solved. These presentations are also convenient when there is no access to a computer lab (see figure 4).



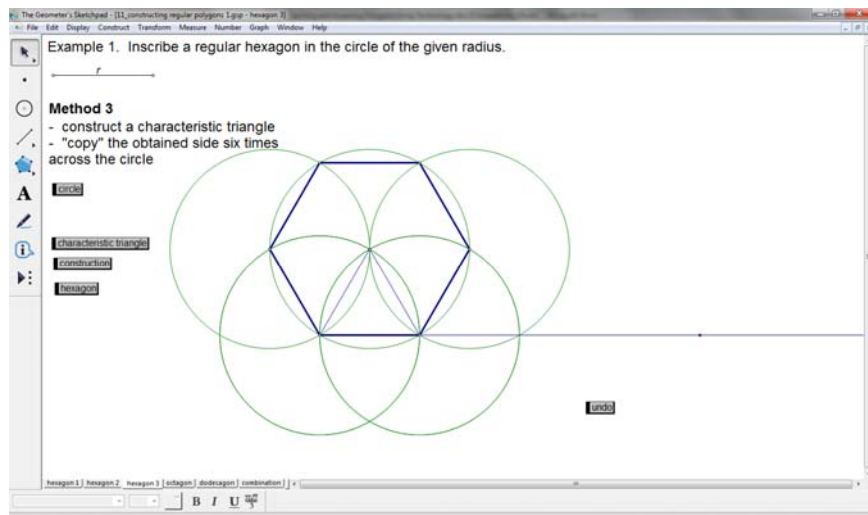
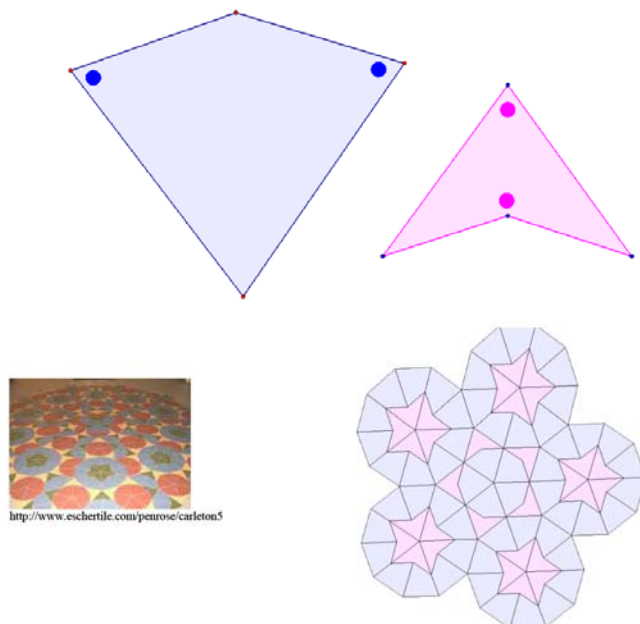


Fig. 4 Step by step presentations showing various methods of inscribing a regular hexagon in the circle of the given radius

Once students have adequate knowledge and understanding of polygons, they can begin to work on various cross-curricular tasks. In addition to building new geometrical ideas, such tasks allow student to see the application of mathematics in real-life and help them develop an appreciation for the subject. Here is an example of one of the cross-curricular tasks we use in this unit.

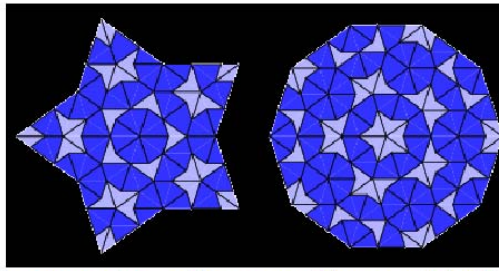
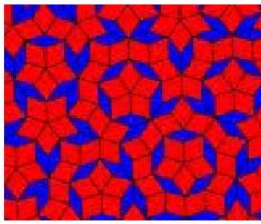
### Penrose tiling

British scientist Sir Roger Penrose came up with a special tiling that uses two shapes, a kite and a dart. The tiles must be placed so that each vertex with a dot always touches only other vertices with dot. These tiles make a non-periodic tiling, and do not repeat by translations.



Penrose tiling created in Sketchpad

Copy the two tiles shown below – the kite and the dart with their dots – on „Sketchpad paper“. Create your unique Penrose tiling. Answer the questions below:

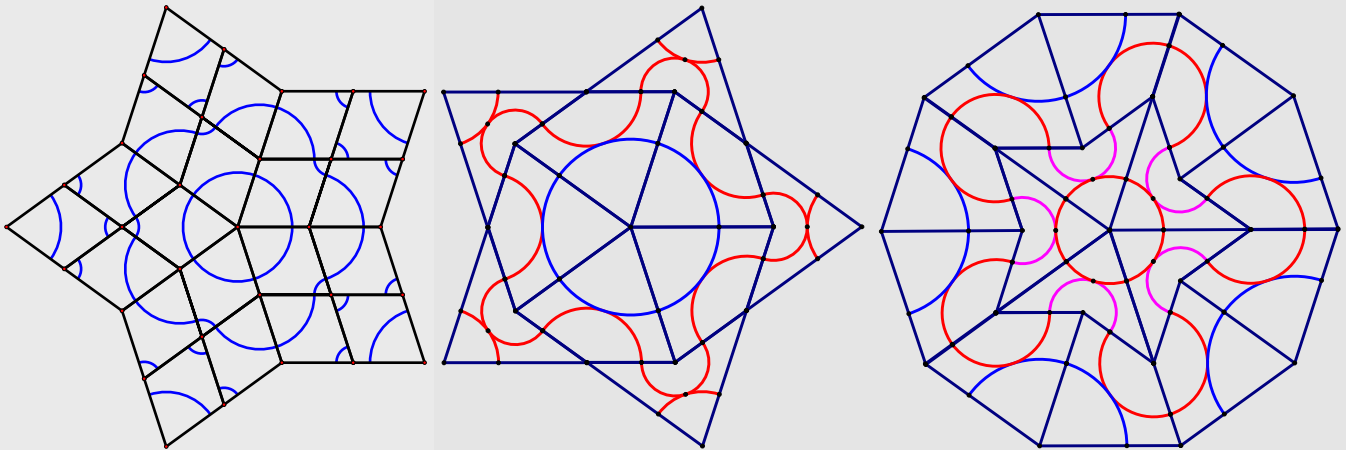


<http://mathworld.wolfram.com/Penrose/11.05.2010/>

1. How many different combinations of kites and darts can surround a vertex point?
2. Explore how the dots affect the tiling. Why the dots are important (see figure 5)?

(Serra, 2008)

Fig. 5 Samples of student work



Technology enables us to individualize instruction because students are able to work at their own pace. They can revisit some previous activities for quick reminders or explore certain concepts more, if they need to. While some students need extra time to complete the assigned tasks, others will be done quicker. Enrichment activities can be offered to those students who complete their task successfully (see figure 6).

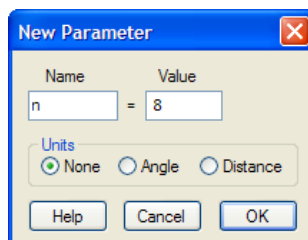
## Enrichment activity – Iteration

### Example 1

Using iterations, construct a regular polygon with the given length of a side.

Solution:

1. Define parameter  $n$  (Use the *Number* menu to select the command *New Parameter.*).

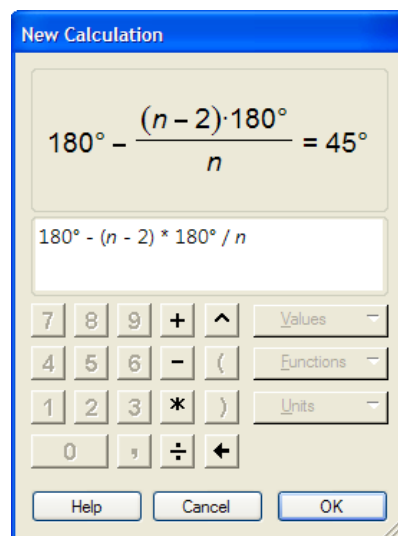


2. Construct segment  $\overline{AB}$ .

3. Calculate the exterior angle of the regular  $n$  – gon  $\left(180^\circ - \frac{(n-2) \cdot 180^\circ}{n}\right)$

Use the *Number* menu to select the command *Calculate* and write as in the picture.

$$180^\circ - \frac{(n-2) \cdot 180^\circ}{n}$$



Note: For  $n$  you select (click on) the parameter  $n$  that you constructed in step 1.

4. Translate point  $A$  for vector  $BA$ .

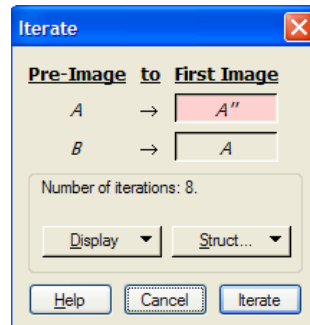
(First select point  $B$  and then point  $A$ . Use the *Transform* menu to select the command *Mark vector*. Next, select point  $A$  and use the *Transform* menu to select the command *Translate*. Name the new point  $A'$ .)

5. Rotate point  $A'$  around point  $A$  by the angle calculated in step 3.

(Double-click on point  $A$ , then select the angle calculated in step 3. Use the *Transform* menu to select the command *Mark angle*. Select point  $A'$  and use the *Transform* menu to select the command *Rotate*. Name the new point  $A''$ .)

6. Hide point  $A'$ . (Use the *Display* menu to select the command *Hide Point*.)

7. Select points  $A$  and  $B$  and parameter  $n$ . Hold the shift key and use the *Transform* menu to select the command *Iterate to Depth*. In the new window, select the image for points  $A$  and  $B$ .



For point  $A$  click on point  $A''$  and for point  $B$  click on point  $B$ . Finish by clicking on *Iterate*.

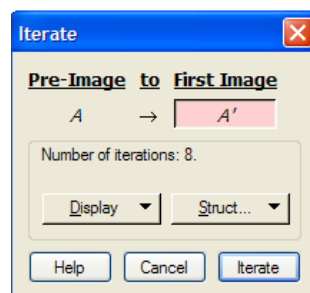
8. Click + or – on parameter  $n$  to change the type of regular polygon.

### Example 2

Using iterations, construct a regular polygon if the radius of a circumscribed circle is given.

#### Solution:

1. Define parameter  $n$  (Use the *Number* menu to select the command *New Parameter*.)
2. Construct a circle and hide the point on the circle.
3. Construct a new point on the circle and label it  $A$  (Select the circle and use the *Construct* menu to select the command *Point on Circle*.)
4. Calculate the central angle of the regular  $n$  – gon  $\left(\frac{360^\circ}{n}\right)$ .
5. Rotate point  $A$  around point  $S$  by the angle calculated in step 4 and label the new point  $A'$ .
6. Draw segment  $\overline{AA'}$ .
7. Select point  $A$  and parameter  $n$ . Hold the shift key and use the *Transform* menu to select the command *Iterate to Depth*. In the new window, select point  $A'$  for the image of point  $A$ .



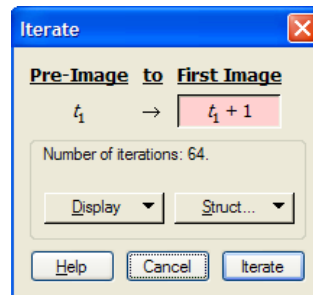
8. Click + or – on parameter  $n$  to change the type of regular polygon.

### Example 3

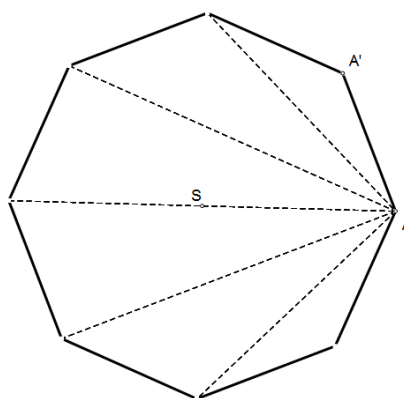
Using iterations, construct a regular polygon and all its diagonals from one vertex.

#### Solution:

1. Construct parameters  $n$  i  $t_1$ . (Parameter  $n$  represents the number of vertices. We need the parameter  $t_1$  for calculations. The value of parameter  $t_1$  is 0.)
2. Draw points  $S$  and  $A$ .
3. Calculate  $n \cdot n$ .
4. Calculate  $t_1 + 1$ .
5. Calculate  $\left(\frac{360^\circ}{n}\right) \cdot (t_1 + 1)$ .
6. Rotate point  $A$  around point  $S$  by the angle  $\left(\frac{360^\circ}{n}\right) \cdot (t_1 + 1)$ . Name the new point  $A'$ .
7. Draw segment  $\overline{AA'}$ .
8. Select point  $A$  and parameter  $n$ . Hold the shift key and use the *Transform* menu to select the command *Iterate to Depth*. In the new window, select point  $A'$  for the image of point  $A$ .
9. Select parameters  $t_1$  i  $n \cdot n$ . Hold the shift key and use the *Transform* menu to select the command *Iterate to Depth*. In the new window, select calculation  $t_1 + 1$  for the image of the parameter  $t_1$ .



10. Click + or – on parameter  $n$  to change the type of regular polygon.





**Example 4**

Using iterations, construct a regular polygon and all its diagonals.

Solution:

1. Draw points  $S$  and  $A$ .
2. Construct parameters  $n$  and  $t_1$ . (The value for parameter  $t_1$  is 0.)
3. Calculate  $n \cdot n$ .
4. Calculate  $t_1 + 1$ .
5. Calculate  $\left(\frac{(t+t_1)}{n}\right) \cdot \left(\frac{360^\circ}{n}\right)$ .
6. Calculate  $\left(\frac{360^\circ}{n}\right) \cdot (t+t_1)$ .
7. Create the Action button *Hide/Show* to hide/show point  $A$ .

(Select point  $A$  and use the *Edit* menu to select the command *Action Buttons* and then the command *Hide/Show*.)

8. Rotate point  $A$  around point  $S$  by the angle  $\text{trunc}\left(\frac{(t+t_1)}{n}\right) \cdot \left(\frac{360^\circ}{n}\right)$ . Label the new point  $A'$ .
9. Using the Action button *Hide/Show*, hide point  $A$ .
10. Insert the Action button *Hide/Show* to hide/show point  $A'$ .
11. Using the Action button *Hide/Show*, hide point  $A'$ .
12. Using the Action button *Hide/Show*, show point  $A$ .
13. Rotate point  $A$  around point  $S$  by the angle  $\left(\frac{360^\circ}{n}\right) \cdot (t+t_1)$ . Label the new point  $A''$ .
14. Using the Action button *Hide/Show*, hide point  $A$  and show point  $A'$ .
15. Draw segment  $\overline{A'A''}$ .
16. Using the Action button *Hide/Show*, hide point  $A'$  and show point  $A$ .
17. Select parameters  $t_1$  and  $n \cdot n$ . Hold the shift key and use the *Transform* menu to select the command *Iterate to Depth*. In the new window, select calculation  $t_1 + 1$  for the image of parameter  $t_1$ .
18. Click + or - on parameter  $n$  to change the type of regular polygon.

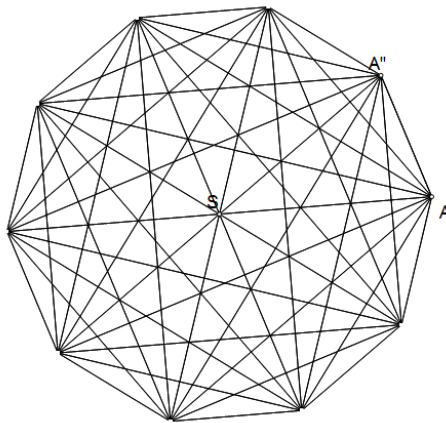
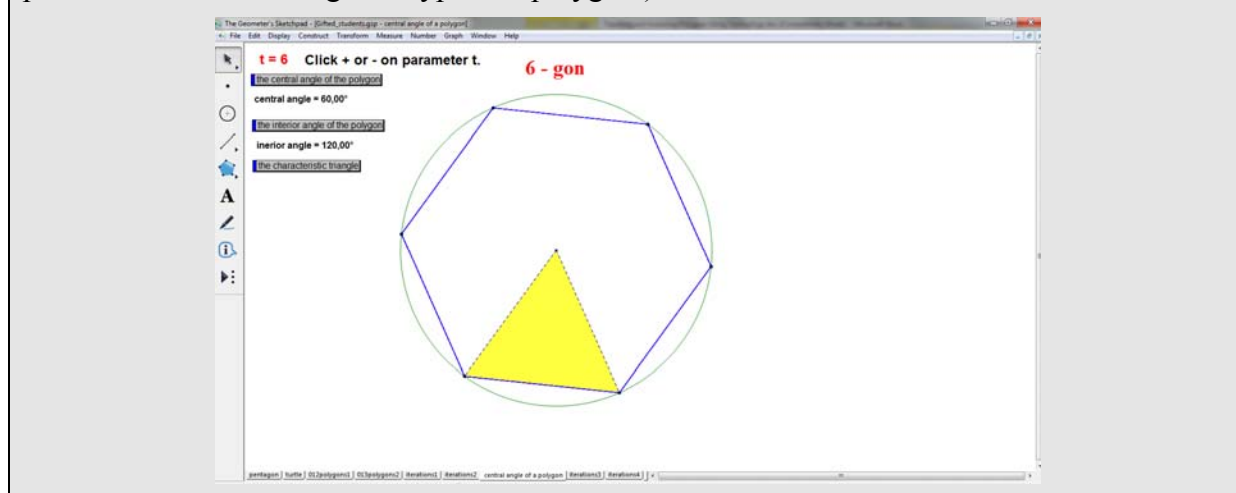


Fig. 6 An example of student work on enrichment activity *Iterations* (By clicking + or – on parameter one can change the type of a polygon.)



The activities used in our model create a motivating and engaging environment where technology allows students to discover mathematics on their own and construct their own understanding. The use of technology enables students to easily notice important properties of objects, work at their own pace, focus on important concepts and build geometrical knowledge. In addition, the use of technology enables teachers to actively involve students in the learning process and individualize instruction. Therefore, it improves the teaching and learning of geometry.

## References

Serra, M. (2008). *Discovering Geometry An Investigate Approach*. Key Curriculum Press, Emeryville, CA.

de Villiers, M. (2003). *Rethinking Proof with The Geometer's Sketchpad*. Key Curriculum Press, Emeryville, CA.

*101 Project Ideas for The Geometer's Sketchpad*. Key Curriculum Press. Available:  
[http://www.dynamicgeometry.com/General\\_Resources/101\\_Project\\_Ideas.html](http://www.dynamicgeometry.com/General_Resources/101_Project_Ideas.html) (19/09/2010)