

# Electronic devices and Linear Algebra

## 1 The problem

Problem:

A surveillance device has access to images from security CCTV that focuses on the four sides of a building.

The device is programmed in such a way that only shows one of the sides. After showing the same side for one minute it may "choose" to maintain the same image, with probability  $a$  ( $a$  between 0 and 1) or may access one of the two adjacent sides of the building, with equal probability  $((1-a)/2)$ .

The security guard controlling the device introduces the value of  $a$ , as a data.

i) Which value of  $a$  should be introduced to display the same side constantly?(or to change always the controlled side?)

ii) At 8:00 a.m the device displays the Nord façade. The guard introduce the value  $a = 1/2$ .

Find the probability of showing each of the facades at 9 am. Analyze the same problem with different values of parameter  $a$ . Pay special attention to the cases  $a=0$  and  $a=1$ .

iii) Study, for different values of the parameter  $a$ , the behavior of the device when  $n$  minutes have passed, with  $n$  very large.

iv) "Repeat" the experiment if you have the same device in a hexagonal square.

v) Would it be possible to draw any conclusions for the position limit for a device located in a polygon with  $h$  sides?.

## 2

### 2.1 Modelling. Theoretical

Analyzing the statement

Identifying data and objectives

Defining variables

Choosing notation

Looking for similar examples in references

Proposing and validating the model

Defining the state in the time  $n$   $[x_1(n), x_2(n), x_3(n), x_4(n)]$ , the transition matrix and to check the model.

### 2.2 Concepts to be used.

An outline of the process, with concepts to be used.

To introduce the matrix  $M$ , to compute its eigenvalues and eigenvectors, to find transition matrix, diagonal matrix and the  $P$  matrix and to calculate the powers of  $M$ .

## 3 Resolution

□ **3.1**

☑ Introducing the matrix

```
(%i1) M: matrix(
      [a,(1-a)/2,0,(1-a)/2],
      [(1-a)/2,a,(1-a)/2,0],
      [0,(1-a)/2,a,(1-a)/2],
      [(1-a)/2,0,(1-a)/2,a]
    );
```

(%o1)

$$\begin{bmatrix} a & \frac{1-a}{2} & 0 & \frac{1-a}{2} \\ \frac{1-a}{2} & a & \frac{1-a}{2} & 0 \\ 0 & \frac{1-a}{2} & a & \frac{1-a}{2} \\ \frac{1-a}{2} & 0 & \frac{1-a}{2} & a \end{bmatrix}$$

☑ (x1,x2,x3,x4) probability vector in the sides (N, E, S,W).

8:00 a. m.: The vector is v= [1,0,0,0].

```
(%i2) v:matrix([1], [0], [0], [0]);
```

(%o2)

$$\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

☑ After one minute the probabilities are M.v.

```
(%i3) M.v;
```

(%o3)

$$\begin{bmatrix} a \\ \frac{1-a}{2} \\ 0 \\ \frac{1-a}{2} \end{bmatrix}$$

☑ N with probability a, E prob. (1-a)/2, S prob 0 and W prob (1-a)/2.

☑ ii) After one hour we have to compute

```
☑ --> M^^60;
```

With general values of a MAXIMA is "exhausted". (Then we will need to diagonalize).

With a=1/2 directly.

```
(%i4) M2: subst(1/2, a, M);
```

```
(%o4) [ 1 1 0 1
        2 4 0 4
        1 1 1 0
        4 2 4 0
        0 1 1 1
        4 2 4 4
        1 1 1 1
        4 0 4 2 ]
```

```
(%i5) M2^^60;
```

```
(%o5) [ 576460752303423489 1 576460752303423487 1
        2305843009213693952 4 2305843009213693952 4
        1 576460752303423489 1 576460752303423487
        4 2305843009213693952 4 2305843009213693952
        576460752303423487 1 576460752303423489 1
        2305843009213693952 4 2305843009213693952 4
        1 576460752303423487 1 576460752303423489
        4 2305843009213693952 4 2305843009213693952 ]
```

```
(%i6) float(%.v);
```

```
(%o6) [ 0.25
        0.25
        0.25
        0.25 ]
```

After 1 hour is more or less with equal probability in the four sides.  
 For arbitrary a and n steps we find the diagonal form (eigenvalues, eigenvectors, P matrix, etc.)

```
(%i7) eigenvalues(M);
```

```
(%o7) [[ 2 a - 1, 1, a ], [ 1, 1, 2 ]]
```

Diagonal form

```
(%i8) D: matrix(
      [2*a-1,0,0,0],
      [0,1,0,0],
      [0,0,a,0],
      [0,0,0,a]
    );
```

(%o8) 
$$\begin{bmatrix} 2a-1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & a & 0 \\ 0 & 0 & 0 & a \end{bmatrix}$$

```
(%i9) eigenvectors(M);
```

```
(%o9) [[ [2 a-1, 1, a], [1, 1, 2]], [[ [1, -1, 1, -1]], [[1, 1, 1, 1]], [[1, 0, -1, 0], [0, 1, 0, -1]]]]
```

P matrix

```
(%i10) P: matrix(
      [1,1,1,0],
      [-1,1,0,1],
      [1,1,-1,0],
      [-1,1,0,-1]
    );
```

(%o10) 
$$\begin{bmatrix} 1 & 1 & 1 & 0 \\ -1 & 1 & 0 & 1 \\ 1 & 1 & -1 & 0 \\ -1 & 1 & 0 & -1 \end{bmatrix}$$

```
(%i11) P^(-1).M.P;
```

(%o11) 
$$\begin{bmatrix} \frac{2a-1}{2} - \frac{1-2a}{2} & 0 & 0 & 0 \\ \frac{2a-1}{2} + \frac{1-2a}{2} & 1 & 0 & 0 \\ 0 & 0 & a & 0 \\ 0 & 0 & 0 & a \end{bmatrix}$$

Simplifying we again get the diagonal matrix D.

```
(%i12) radcan(%);
```

(%o12) 
$$\begin{bmatrix} 2a-1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & a & 0 \\ 0 & 0 & 0 & a \end{bmatrix}$$

After 1 hour

```
(%i13) D^^60;
```

$$\begin{pmatrix}
 (2a-1)^{60} & 0 & 0 & 0 \\
 0 & 1 & 0 & 0 \\
 0 & 0 & a^{60} & 0 \\
 0 & 0 & 0 & a^{60}
 \end{pmatrix}$$

```
(%i14) step60:P.(D^^60).P^^(-1);
```

$$\begin{pmatrix}
 \frac{(2a-1)^{60}}{4} + \frac{a^{60}}{2} + \frac{1}{4} & \frac{1}{4} - \frac{(2a-1)^{60}}{4} & \frac{(2a-1)^{60}}{4} - \frac{a^{60}}{2} + \frac{1}{4} & \frac{1}{4} - \frac{(2a-1)^{60}}{4} \\
 \frac{1}{4} - \frac{(2a-1)^{60}}{4} & \frac{(2a-1)^{60}}{4} + \frac{a^{60}}{2} + \frac{1}{4} & \frac{1}{4} - \frac{(2a-1)^{60}}{4} & \frac{(2a-1)^{60}}{4} - \frac{a^{60}}{2} + \frac{1}{4} \\
 \frac{(2a-1)^{60}}{4} - \frac{a^{60}}{2} + \frac{1}{4} & \frac{1}{4} - \frac{(2a-1)^{60}}{4} & \frac{(2a-1)^{60}}{4} + \frac{a^{60}}{2} + \frac{1}{4} & \frac{1}{4} - \frac{(2a-1)^{60}}{4} \\
 \frac{1}{4} - \frac{(2a-1)^{60}}{4} & \frac{(2a-1)^{60}}{4} - \frac{a^{60}}{2} + \frac{1}{4} & \frac{1}{4} - \frac{(2a-1)^{60}}{4} & \frac{(2a-1)^{60}}{4} + \frac{a^{60}}{2} + \frac{1}{4}
 \end{pmatrix}$$

iii) For n steps

```
(%i15) stepn:P.D^^n.P^^(-1);
```

$$\begin{pmatrix} 1 & 1 & 1 & 0 \\ -1 & 1 & 0 & 1 \\ 1 & 1 & -1 & 0 \\ -1 & 1 & 0 & -1 \end{pmatrix} \cdot \begin{pmatrix} 2a-1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & a & 0 \\ 0 & 0 & 0 & a \end{pmatrix}^n \cdot \begin{pmatrix} \frac{1}{4} & -\frac{1}{4} & \frac{1}{4} & -\frac{1}{4} \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{2} & 0 & -\frac{1}{2} & 0 \\ 0 & \frac{1}{2} & 0 & -\frac{1}{2} \end{pmatrix}$$

Maxima is not able to compute D^n. Introducing the data by hand (D is diagonal matrix)

```
(%i16) dn:matrix([(2*a-1)^n,0,0,0],[0,1,0,0],[0,0,a^n,0],[0,0,0,a^n]);
```

$$\begin{pmatrix}
 (2a-1)^n & 0 & 0 & 0 \\
 0 & 1 & 0 & 0 \\
 0 & 0 & a^n & 0 \\
 0 & 0 & 0 & a^n
 \end{pmatrix}$$

```
(%i17) sn:P. dn .P^(-1);
```

$$\begin{bmatrix}
 \frac{(2a-1)^n}{4} + \frac{a^n}{2} + \frac{1}{4} & \frac{1}{4} - \frac{(2a-1)^n}{4} & \frac{(2a-1)^n}{4} - \frac{a^n}{2} + \frac{1}{4} & \frac{1}{4} - \frac{(2a-1)^n}{4} \\
 \frac{1}{4} - \frac{(2a-1)^n}{4} & \frac{(2a-1)^n}{4} + \frac{a^n}{2} + \frac{1}{4} & \frac{1}{4} - \frac{(2a-1)^n}{4} & \frac{(2a-1)^n}{4} - \frac{a^n}{2} + \frac{1}{4} \\
 \frac{(2a-1)^n}{4} - \frac{a^n}{2} + \frac{1}{4} & \frac{1}{4} - \frac{(2a-1)^n}{4} & \frac{(2a-1)^n}{4} + \frac{a^n}{2} + \frac{1}{4} & \frac{1}{4} - \frac{(2a-1)^n}{4} \\
 \frac{1}{4} - \frac{(2a-1)^n}{4} & \frac{(2a-1)^n}{4} - \frac{a^n}{2} + \frac{1}{4} & \frac{1}{4} - \frac{(2a-1)^n}{4} & \frac{(2a-1)^n}{4} + \frac{a^n}{2} + \frac{1}{4}
 \end{bmatrix}$$

```
(%o17)
```

```
(%i18) sn2:sn,a=1/2;
```

$$\begin{bmatrix}
 2^{-n-1} + \frac{1}{4} & \frac{1}{4} & \frac{1}{4} - 2^{-n-1} & \frac{1}{4} \\
 \frac{1}{4} & 2^{-n-1} + \frac{1}{4} & \frac{1}{4} & \frac{1}{4} - 2^{-n-1} \\
 \frac{1}{4} - 2^{-n-1} & \frac{1}{4} & 2^{-n-1} + \frac{1}{4} & \frac{1}{4} \\
 \frac{1}{4} & \frac{1}{4} - 2^{-n-1} & \frac{1}{4} & 2^{-n-1} + \frac{1}{4}
 \end{bmatrix}$$

```
(%o18)
```

```
(%i19) limit(sn2, n, inf);
```

$$\begin{bmatrix}
 \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\
 \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\
 \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\
 \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4}
 \end{bmatrix}$$

```
(%o19)
```

```
(%i20) limit(sn2.v, n, inf);
```

$$\begin{bmatrix}
 \frac{1}{4} \\
 \frac{1}{4} \\
 \frac{1}{4} \\
 \frac{1}{4}
 \end{bmatrix}$$

```
(%o20)
```

```
(%i21) sn1:sn,a=1;
```

$$(\%o21) \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

With a=1, Identity\_Matrix

With a=0:

```
(%i22) sn0:sn,a=0;
```

$$(\%o22) \begin{bmatrix} \frac{(-1)^n + 1}{4} & \frac{1 - (-1)^n}{4} & \frac{1 - (-1)^n}{4} & \frac{(-1)^n + 1}{4} & \frac{1 - (-1)^n}{4} & \frac{1 - (-1)^n}{4} \\ \frac{1 - (-1)^n}{4} & \frac{(-1)^n + 1}{4} & \frac{(-1)^n + 1}{4} & \frac{1 - (-1)^n}{4} & \frac{1 - (-1)^n}{4} & \frac{(-1)^n + 1}{4} \\ \frac{(-1)^n + 1}{4} & \frac{1 - (-1)^n}{4} & \frac{1 - (-1)^n}{4} & \frac{(-1)^n + 1}{4} & \frac{1 - (-1)^n}{4} & \frac{1 - (-1)^n}{4} \\ \frac{1 - (-1)^n}{4} & \frac{(-1)^n + 1}{4} & \frac{(-1)^n + 1}{4} & \frac{1 - (-1)^n}{4} & \frac{1 - (-1)^n}{4} & \frac{(-1)^n + 1}{4} \end{bmatrix}$$

```
(%i23) t0:sn0.v;
```

$$(\%o23) \begin{bmatrix} \frac{(-1)^n + 1}{4} \\ \frac{1 - (-1)^n}{4} \\ \frac{(-1)^n + 1}{4} \\ \frac{1 - (-1)^n}{4} \end{bmatrix}$$

t0 is different, depending on whether n is even or odd

```
(%i24) t0,n=30;
```

$$(\%o24) \begin{bmatrix} 1 \\ 2 \\ 0 \\ 1 \\ 2 \\ 0 \end{bmatrix}$$

```
(%i25) t0,n=31;
```

(%o25)

$$\begin{bmatrix} 0 \\ \frac{1}{2} \\ 0 \\ \frac{1}{2} \end{bmatrix}$$

```
(%i26) limit(t0, n, inf);
```

(%o26)

$$\begin{bmatrix} ind \\ ind \\ ind \\ ind \end{bmatrix}$$

```
(%i27) tn:sn.v;
```

(%o27)

$$\begin{bmatrix} \frac{(2a-1)^n}{4} + \frac{a^n}{2} + \frac{1}{4} \\ \frac{1}{4} - \frac{(2a-1)^n}{4} \\ \frac{(2a-1)^n}{4} - \frac{a^n}{2} + \frac{1}{4} \\ \frac{1}{4} - \frac{(2a-1)^n}{4} \end{bmatrix}$$

```
(%i28) limit(tn,n,inf);
```

(%o28)

$$\begin{bmatrix} \lim_{n \rightarrow \infty} \frac{(2a-1)^n}{4} + \frac{a^n}{2} + \frac{1}{4} \\ \lim_{n \rightarrow \infty} \frac{1}{4} - \frac{(2a-1)^n}{4} \\ \lim_{n \rightarrow \infty} \frac{(2a-1)^n}{4} - \frac{a^n}{2} + \frac{1}{4} \\ \lim_{n \rightarrow \infty} \frac{1}{4} - \frac{(2a-1)^n}{4} \end{bmatrix}$$

```
(%i29) t3:tn,a=1/3;
```

$$\begin{aligned}
 & \left[ \begin{array}{l} \frac{(-1)^n}{4 \cdot 3^n} + \frac{1}{2 \cdot 3^n} + \frac{1}{4} \\ \frac{1}{4} - \frac{(-1)^n}{4 \cdot 3^n} \\ \frac{(-1)^n}{4 \cdot 3^n} - \frac{1}{2 \cdot 3^n} + \frac{1}{4} \\ \frac{1}{4} - \frac{(-1)^n}{4 \cdot 3^n} \end{array} \right] \\
 & (\%o29)
 \end{aligned}$$

```
(%i30) limit(t3, n, inf);
```

$$\begin{aligned}
 & \left[ \begin{array}{l} \frac{1}{4} \\ \frac{1}{4} \\ \frac{1}{4} \\ \frac{1}{4} \end{array} \right] \\
 & (\%o30)
 \end{aligned}$$

□ **4 Generalization ( Hexagonal building)**

```
(%i31) kill(all);
```

```
(%o0) done
```

```
(%i1) M: matrix(
    [a,(1-a)/2,0,0,0,(1-a)/2],
    [(1-a)/2,a,(1-a)/2,0,0,0],
    [0,(1-a)/2,a,(1-a)/2,0,0],
    [0,0,(1-a)/2,a,(1-a)/2,0],
    [0,0,0,(1-a)/2,a,(1-a)/2],
    [(1-a)/2,0,0,0,(1-a)/2,a]
);
```

(%o1)

$$\begin{bmatrix} a & \frac{1-a}{2} & 0 & 0 & 0 & \frac{1-a}{2} \\ \frac{1-a}{2} & a & \frac{1-a}{2} & 0 & 0 & 0 \\ 0 & \frac{1-a}{2} & a & \frac{1-a}{2} & 0 & 0 \\ 0 & 0 & \frac{1-a}{2} & a & \frac{1-a}{2} & 0 \\ 0 & 0 & 0 & \frac{1-a}{2} & a & \frac{1-a}{2} \\ \frac{1-a}{2} & 0 & 0 & 0 & \frac{1-a}{2} & a \end{bmatrix}$$

```
(%i2) v:matrix([1], [0], [0], [0],[0],[0]);
```

(%o2)

$$\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

```
(%i3) M.v;
```

(%o3)

$$\begin{bmatrix} a \\ \frac{1-a}{2} \\ 0 \\ 0 \\ 0 \\ \frac{1-a}{2} \end{bmatrix}$$

2 minutes later

```
(%i4) (M^2).v;
```

$$\begin{bmatrix} a^2 + \frac{(1-a)^2}{2} \\ (1-a)a \\ \frac{(1-a)^2}{4} \\ 0 \\ \frac{(1-a)^2}{4} \\ (1-a)a \end{bmatrix}$$

```
(%o4)
```

i) Wiht a=1. M=Identitiy\_Matrix

ii) Afther an hour M^60.v.

With a=1/2:

```
(%i5) M2: M, a=1/2;
```

$$\begin{bmatrix} \frac{1}{2} & \frac{1}{4} & 0 & 0 & 0 & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{2} & \frac{1}{4} & 0 & 0 & 0 \\ 0 & \frac{1}{4} & \frac{1}{2} & \frac{1}{4} & 0 & 0 \\ 0 & 0 & \frac{1}{4} & \frac{1}{2} & \frac{1}{4} & 0 \\ 0 & 0 & 0 & \frac{1}{4} & \frac{1}{2} & \frac{1}{4} \\ \frac{1}{4} & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{2} \end{bmatrix}$$

```
(%o5)
```

```
(%i6) float((M2^60).v);
```

$$\begin{bmatrix} 0.16666667729719 \\ 0.16666667198193 \\ 0.16666666135141 \\ 0.16666665603615 \\ 0.16666666135141 \\ 0.16666667198193 \end{bmatrix}$$

```
(%o6)
```

The probability is the same for all the sides

```
(%i7) eigenvalues(M);
(%o7) [[2 a-1, 1,  $\frac{a+1}{2}$ ,  $\frac{3 a-1}{2}$ ], [1, 1, 2, 2]]
```

```
(%i8) D: matrix(
    [2*a-1,0,0,0,0,0],
    [0,1,0,0,0,0],
    [0,0,(a+1)/2,0,0,0],
    [0,0,0,(a+1)/2,0,0],
    [0,0,0,0,(3*a-1)/2,0],
    [0,0,0,0,0,(3*a-1)/2]
);
(%o8) [
    2 a-1 0 0 0 0 0
    0 1 0 0 0 0
    0 0  $\frac{a+1}{2}$  0 0 0
    0 0 0  $\frac{a+1}{2}$  0 0
    0 0 0 0  $\frac{3 a-1}{2}$  0
    0 0 0 0 0  $\frac{3 a-1}{2}$ 
]
```

```
(%i9) eigenvectors(M);
(%o9) [[[[2 a-1, 1,  $\frac{a+1}{2}$ ,  $\frac{3 a-1}{2}$ ], [1, 1, 2, 2]], [[1, -1, 1, -1, 1, -1]], [
[1, 1, 1, 1, 1, 1]], [[1, 0, -1, -1, 0, 1], [0, 1, 1, 0, -1, -1]], [[1, 0, -1,
1, 0, -1], [0, 1, -1, 0, 1, -1]]]]
```

```
(%i10) P: transpose(matrix([1,-1,1,-1,1,-1], [1,1,1,1,1,1],
    [1,0,-1,-1,0,1],[0,1,1,0,-1,-1],
    [1,0,-1,1,0,-1],[0,1,-1,0,1,-1]));
(%o10) [
    1 1 1 0 1 0
    -1 1 0 1 0 1
    1 1 -1 1 -1 -1
    -1 1 -1 0 1 0
    1 1 0 -1 0 1
    -1 1 1 -1 -1 -1
]
```

```
(%i11) P^(-1).M.P;
(%o11)
```

$$\begin{bmatrix} \frac{2a-1}{2} - \frac{1-2a}{2} & 0 & 0 & 0 & 0 & 0 \\ \frac{2a-1}{2} + \frac{1-2a}{2} & 1 & \frac{a + \frac{1-a}{2} - a - \frac{1-a}{2}}{3} + \frac{1-a}{3} & \frac{a + \frac{1-a}{2} - a - \frac{1-a}{2}}{3} + \frac{1-a}{3} & \frac{a - \frac{1-a}{2} - \frac{1-a}{2} - a}{3} + \frac{1-a}{3} & \frac{a - \frac{1-a}{2} - \frac{1-a}{2} - a}{3} + \frac{1-a}{3} \\ 0 & 0 & \frac{a + \frac{1-a}{2} - a - \frac{1-a}{2}}{2} - \frac{1-a}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{a + \frac{1-a}{2} - a - \frac{1-a}{2}}{2} - \frac{1-a}{2} & 0 & 0 \\ 0 & 0 & \frac{a + \frac{1-a}{2} - a - \frac{1-a}{2}}{6} + \frac{1-a}{6} & -\frac{a + \frac{1-a}{2} - a - \frac{1-a}{2}}{3} - \frac{1-a}{3} & \frac{2\left(a - \frac{1-a}{2}\right) - \frac{1-a}{2} - a}{3} & -\frac{a - \frac{1-a}{2} - \frac{1-a}{2} - a}{3} - \frac{1-a}{3} \\ 0 & 0 & \frac{a + \frac{1-a}{2} - a - \frac{1-a}{2}}{3} - \frac{1-a}{3} & \frac{a + \frac{1-a}{2} - a - \frac{1-a}{2}}{6} + \frac{1-a}{6} & -\frac{a - \frac{1-a}{2} - \frac{1-a}{2} - a}{3} - \frac{1-a}{3} & \frac{2\left(a - \frac{1-a}{2}\right) - \frac{1-a}{2} - a}{3} - \frac{1-a}{3} \end{bmatrix}$$

Simplifying we get D.

```
(%i12) radcan(%);
(%o12)
```

$$\begin{bmatrix} 2a-1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{a+1}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{a+1}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{3a-1}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{3a-1}{2} \end{bmatrix}$$

For computing M^n we do:

```
(%i13) dn:matrix([(2*a-1)^n,0,0,0,0,0],[0,1,0,0,0,0],
[0,0,((a+1)/2)^n,0,0,0],[0,0,0,((a+1)/2)^n,0,0],
[0,0,0,0,((3*a-1)/2)^n,0],[0,0,0,0,0,((3*a-1)/2)^n]);
```

(%o13)

$$\begin{bmatrix} (2a-1)^n & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{(a+1)^n}{2^n} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{(a+1)^n}{2^n} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{(3a-1)^n}{2^n} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{(3a-1)^n}{2^n} \end{bmatrix}$$

```
(%i14) mn:P.(dn).P^(-1);
```

(%o14)

$$\begin{aligned} & \frac{(3a-1)^n}{3 \cdot 2^n} + \frac{(a+1)^n}{3 \cdot 2^n} + \frac{(2a-1)^n}{6} + \frac{1}{6} & - \frac{(3a-1)^n 2^{-n-1}}{3} + \frac{(a+1)^n 2^{-n-1}}{3} \\ & - \frac{(3a-1)^n 2^{-n-1}}{3} + \frac{(a+1)^n 2^{-n-1}}{3} - \frac{(2a-1)^n}{6} + \frac{1}{6} & \frac{(3a-1)^n}{3 \cdot 2^n} + \frac{(a+1)^n}{3 \cdot 2^n} + \frac{(2a-1)^n}{6} \\ & - \frac{(3a-1)^n}{3 \cdot 2^n} - \frac{(a+1)^n}{3 \cdot 2^n} + \frac{(3a-1)^n 2^{-n-1}}{3} + \frac{(a+1)^n 2^{-n-1}}{3} + \frac{(2a-1)^n}{6} + \frac{1}{6} & - \frac{(3a-1)^n}{3 \cdot 2^n} + \frac{(a+1)^n}{3 \cdot 2^n} + \frac{(3a-1)^n 2^{-n-1}}{3} - \frac{(a+1)^n 2^{-n-1}}{3} \\ & \frac{(3a-1)^n}{3 \cdot 2^n} - \frac{(a+1)^n}{3 \cdot 2^n} - \frac{(2a-1)^n}{6} + \frac{1}{6} & - \frac{(3a-1)^n 2^{-n-1}}{3} - \frac{(a+1)^n 2^{-n-1}}{3} \\ & - \frac{(3a-1)^n 2^{-n-1}}{3} - \frac{(a+1)^n 2^{-n-1}}{3} + \frac{(2a-1)^n}{6} + \frac{1}{6} & \frac{(3a-1)^n}{3 \cdot 2^n} - \frac{(a+1)^n}{3 \cdot 2^n} - \frac{(2a-1)^n}{6} \\ & - \frac{(3a-1)^n}{3 \cdot 2^n} + \frac{(a+1)^n}{3 \cdot 2^n} + \frac{(3a-1)^n 2^{-n-1}}{3} - \frac{(a+1)^n 2^{-n-1}}{3} - \frac{(2a-1)^n}{6} + \frac{1}{6} & - \frac{(3a-1)^n}{3 \cdot 2^n} - \frac{(a+1)^n}{3 \cdot 2^n} + \frac{(3a-1)^n 2^{-n-1}}{3} + \frac{(a+1)^n 2^{-n-1}}{3} \end{aligned}$$

```
(%i15) tn:mn.v;
```

(%o15)

$$\left[ \begin{array}{c} \frac{(3a-1)^n (a+1)^n (2a-1)^n}{3 \cdot 2^n} + \frac{1}{6} \\ - \frac{(3a-1)^n 2^{-n-1}}{3} + \frac{(a+1)^n 2^{-n-1}}{3} - \frac{(2a-1)^n}{6} + \frac{1}{6} \\ - \frac{(3a-1)^n}{3 \cdot 2^n} - \frac{(a+1)^n}{3 \cdot 2^n} + \frac{(3a-1)^n 2^{-n-1}}{3} + \frac{(a+1)^n 2^{-n-1}}{3} + \frac{(2a-1)^n}{6} + \frac{1}{6} \\ \frac{(3a-1)^n (a+1)^n (2a-1)^n}{3 \cdot 2^n} - \frac{1}{6} \\ - \frac{(3a-1)^n 2^{-n-1}}{3} - \frac{(a+1)^n 2^{-n-1}}{3} + \frac{(2a-1)^n}{6} + \frac{1}{6} \\ - \frac{(3a-1)^n}{3 \cdot 2^n} + \frac{(a+1)^n}{3 \cdot 2^n} + \frac{(3a-1)^n 2^{-n-1}}{3} - \frac{(a+1)^n 2^{-n-1}}{3} - \frac{(2a-1)^n}{6} + \frac{1}{6} \end{array} \right]$$

```
(%i16) tn2:tn,a=1/2;
```

(%o16)

$$\left[ \begin{array}{c} \frac{3^{n-1}}{2^{2n}} + \frac{1}{3 \cdot 2^{2n}} + \frac{1}{6} \\ 2^{-2n-1} 3^{n-1} - \frac{2^{-2n-1}}{3} + \frac{1}{6} \\ - \frac{3^{n-1}}{2^{2n}} + 2^{-2n-1} 3^{n-1} - \frac{1}{3 \cdot 2^{2n}} + \frac{2^{-2n-1}}{3} + \frac{1}{6} \\ - \frac{3^{n-1}}{2^{2n}} + \frac{1}{3 \cdot 2^{2n}} + \frac{1}{6} \\ - 2^{-2n-1} 3^{n-1} - \frac{2^{-2n-1}}{3} + \frac{1}{6} \\ \frac{3^{n-1}}{2^{2n}} - 2^{-2n-1} 3^{n-1} - \frac{1}{3 \cdot 2^{2n}} + \frac{2^{-2n-1}}{3} + \frac{1}{6} \end{array} \right]$$

```
(%i17) limit(tn2, n, inf);
```

(%o17)

$$\left[ \begin{array}{c} \frac{1}{6} \\ \frac{1}{6} \\ \frac{1}{6} \\ \frac{1}{6} \\ \frac{1}{6} \\ \frac{1}{6} \end{array} \right]$$

```
(%i18) tn3:tn,a=1/3;
```

$$\begin{bmatrix}
 \frac{3^{-n-1} 4^n}{2^n} + \frac{3^{-n-1} (-1)^n}{2} + \frac{1}{6} \\
 2^{-n-1} 3^{-n-1} 4^n - \frac{3^{-n-1} (-1)^n}{2} + \frac{1}{6} \\
 -\frac{3^{-n-1} 4^n}{2^n} + 2^{-n-1} 3^{-n-1} 4^n + \frac{3^{-n-1} (-1)^n}{2} + \frac{1}{6} \\
 -\frac{3^{-n-1} 4^n}{2^n} - \frac{3^{-n-1} (-1)^n}{2} + \frac{1}{6} \\
 -2^{-n-1} 3^{-n-1} 4^n + \frac{3^{-n-1} (-1)^n}{2} + \frac{1}{6} \\
 \frac{3^{-n-1} 4^n}{2^n} - 2^{-n-1} 3^{-n-1} 4^n - \frac{3^{-n-1} (-1)^n}{2} + \frac{1}{6}
 \end{bmatrix}$$

```
(%o18)
```

```
(%i19) limit(tn3 , n, inf);
```

$$\begin{bmatrix}
 \frac{1}{6} \\
 \frac{1}{6} \\
 \frac{1}{6} \\
 \frac{1}{6} \\
 \frac{1}{6} \\
 \frac{1}{6}
 \end{bmatrix}$$

```
(%o19)
```

```
(%i20) tn1:tn, a=1;
```

$$\begin{bmatrix}
 1 \\
 0 \\
 0 \\
 0 \\
 0 \\
 0
 \end{bmatrix}$$

```
(%o20)
```

```
(%i21) tn0:tn, a=0;
```

$$\left[ \begin{array}{c}
 \frac{(-1)^n}{3 \cdot 2^n} + \frac{(-1)^n}{6} + \frac{1}{3 \cdot 2^n} + \frac{1}{6} \\
 - \frac{2^{-n-1}(-1)^n}{3} - \frac{(-1)^n}{6} + \frac{2^{-n-1}}{3} + \frac{1}{6} \\
 - \frac{(-1)^n}{3 \cdot 2^n} + \frac{2^{-n-1}(-1)^n}{3} + \frac{(-1)^n}{6} - \frac{1}{3 \cdot 2^n} + \frac{2^{-n-1}}{3} + \frac{1}{6} \\
 \frac{(-1)^n}{3 \cdot 2^n} - \frac{(-1)^n}{6} - \frac{1}{3 \cdot 2^n} + \frac{1}{6} \\
 - \frac{2^{-n-1}(-1)^n}{3} + \frac{(-1)^n}{6} - \frac{2^{-n-1}}{3} + \frac{1}{6} \\
 - \frac{(-1)^n}{3 \cdot 2^n} + \frac{2^{-n-1}(-1)^n}{3} - \frac{(-1)^n}{6} + \frac{1}{3 \cdot 2^n} - \frac{2^{-n-1}}{3} + \frac{1}{6}
 \end{array} \right]$$

```
(%o21)
```

```
(%i22) tn036:tn0,n=36;
```

$$\left[ \begin{array}{c}
 \frac{11453246123}{34359738368} \\
 0 \\
 \frac{22906492245}{68719476736} \\
 0 \\
 \frac{22906492245}{68719476736} \\
 0
 \end{array} \right]$$

```
(%o22)
```

```
(%i23) float(%);
```

$$\left[ \begin{array}{c}
 0.333333333334303 \\
 0.0 \\
 0.333333333332848 \\
 0.0 \\
 0.333333333332848 \\
 0.0
 \end{array} \right]$$

```
(%o23)
```

```
(%i24) tn037:tn0,n=37;
      0
      45812984491
      137438953472
      0
(%o24) 22906492245
      68719476736
      0
      45812984491
      137438953472
```

```
(%i25) float(%);
      0.0
      0.333333333333576
      0.0
(%o25) 0.333333333332848
      0.0
      0.333333333333576
```

Then for a different from 0 and 1 the probability is the same 1/6 for each side.  
 For a=1 the side is always the same.  
 For a=0 in the even and odd steps the behavior is completely different.

**5 For h sides**

Hint: The matrix will be hXh.  
 The behavior is COMPLETELY DIFFERENT in the cases h odd or even.