

# Base Technologies for Tutoring

## Experiences from experiments in the *ISAC*-project

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- 1 Remarks on tutoring software
  - Status quo in the design
  - Requirements on the design
- 2 Base technologies for tutoring
  - Computer theorem proving
  - Single-stepping program interpretation
  - Human readable math knowledge
- 3 Summary and Conclusion

# Outline

- 1 Remarks on tutoring software  
Status quo in the design  
Requirements on the design
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Computer theorem proving  
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# Status quo in tutoring software

Software for tutoring is concerned with *individuals* ...

- ... *individual learners*
  - on different levels
  - with different pace in learning ...
- ... *individual teachers*
  - with different teaching styles
  - emphasizing specific examples ...
- ... *individual programmers* (frequently teachers)
  - creating an abundant variety of software
  - lack support of general software services

What are the **general requirements** for tutoring ?

What are the **basic technologies** to meet the requirements ?

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# Requirements for tutoring

Tutoring

Status quo

Requirements

Technologies

CTP

Debug mode

Transparent

Summary and  
Conclusion

The general requirements are basically:

- 1 **Check user input** as generous as possible ...
- 2 **Guide the user** step by step towards a solution ...
- 3 **Explain steps** on request by the user ...

... during the stepwise *construction* of the solution  
of some problem in applying mathematics (incl. geometry).

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# Demo Isabelle from wikipediaedia

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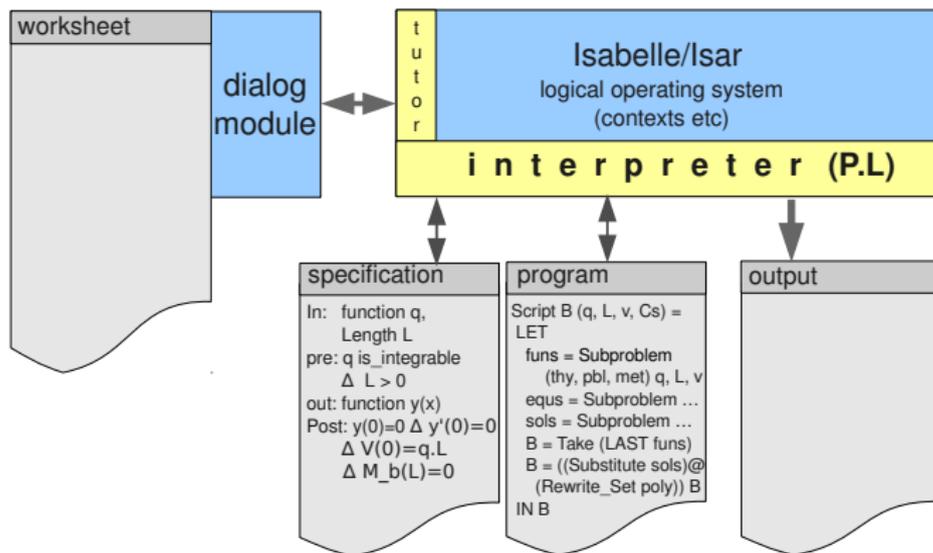
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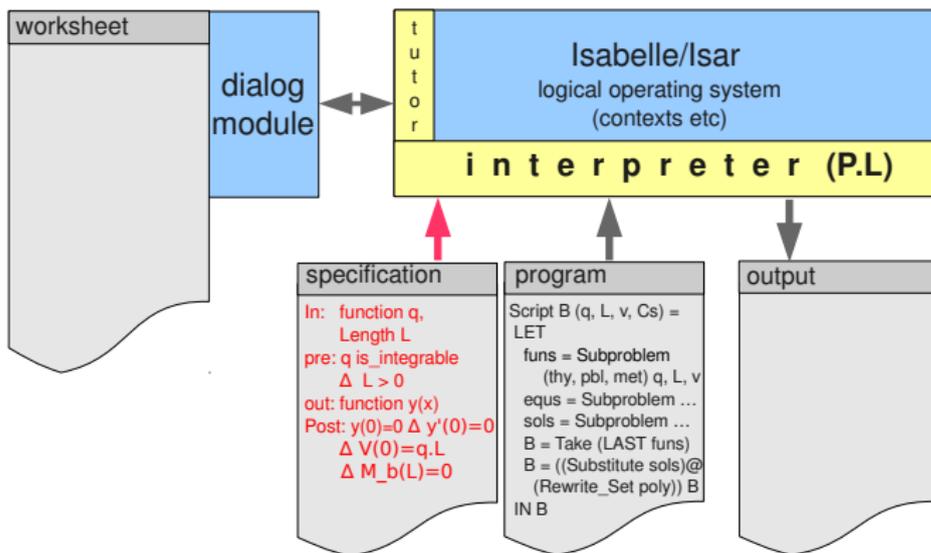
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# Interpretation in debug-mode



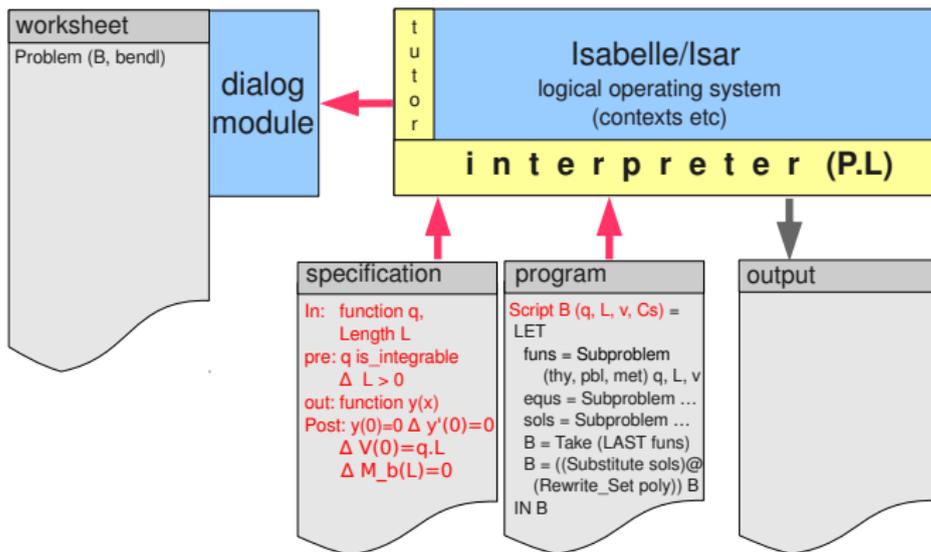
Authoring

# Example :start tutoring



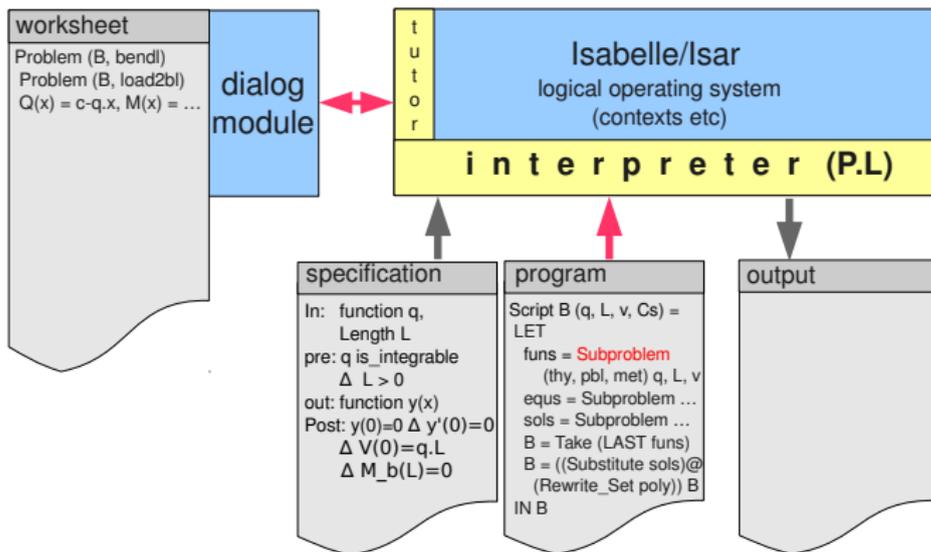
Tutoring

# Example: Tutoring start



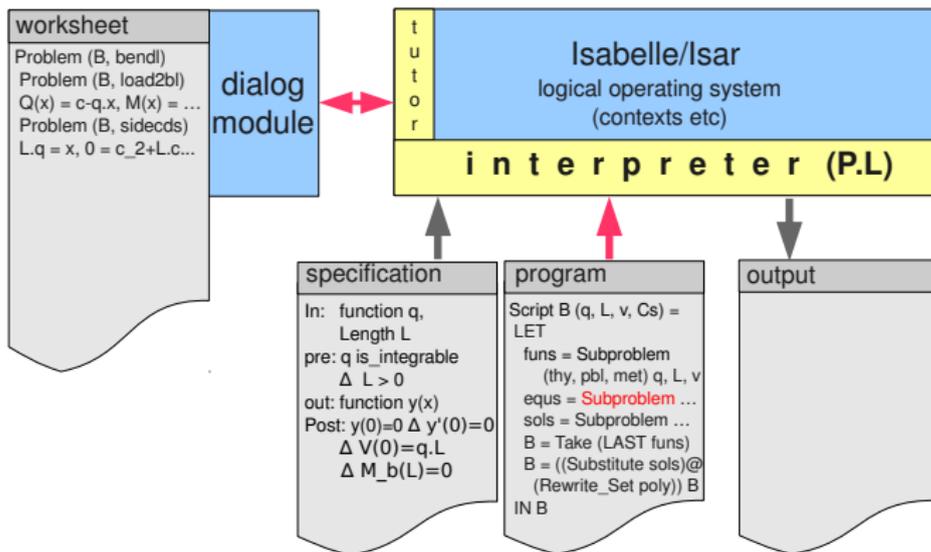
Tutoring

# Example: Tutoring step 1



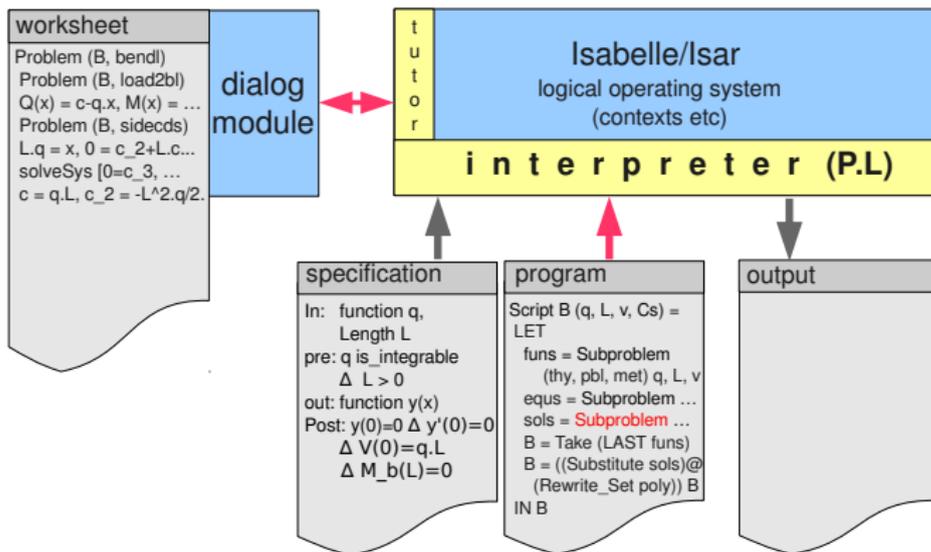
Tutoring

## Example: Tutoring step 2



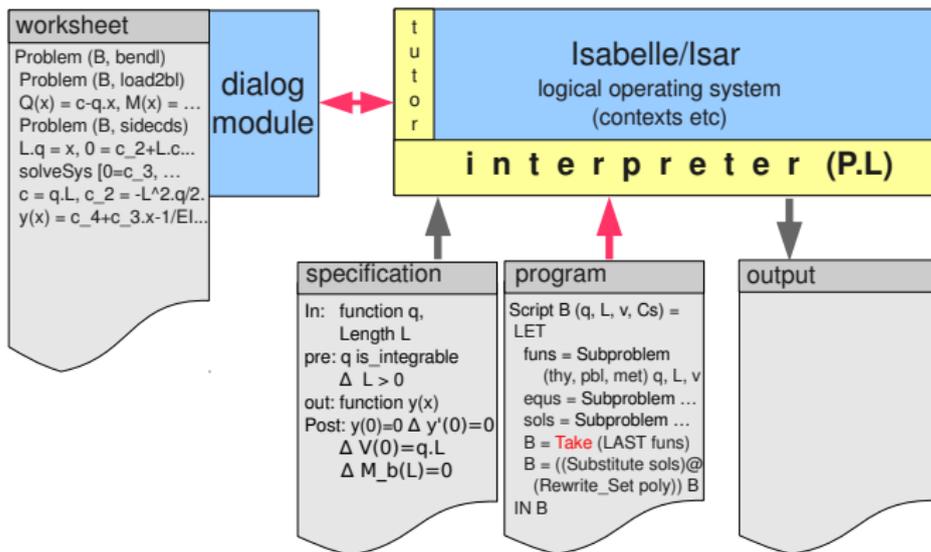
Tutoring

# Example: Tutoring step 3



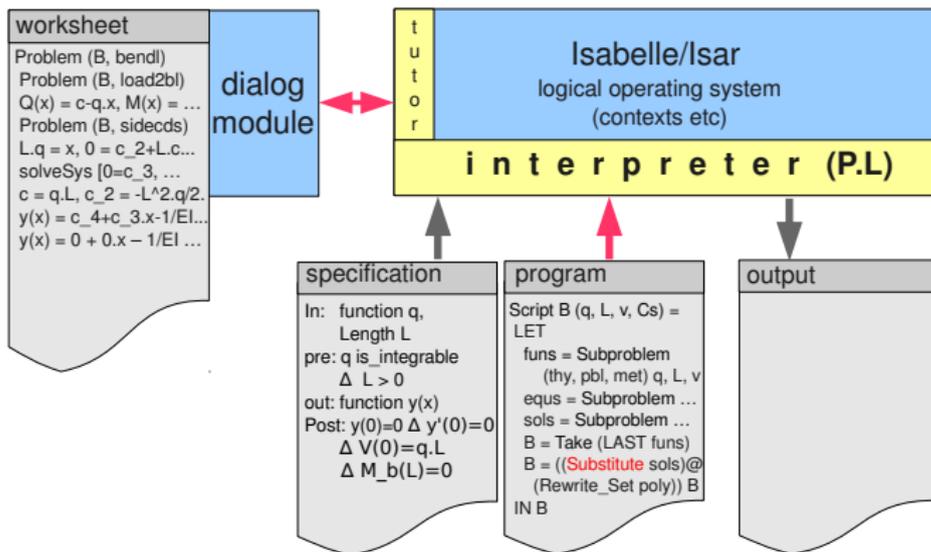
Tutoring

# Example: Tutoring step 4



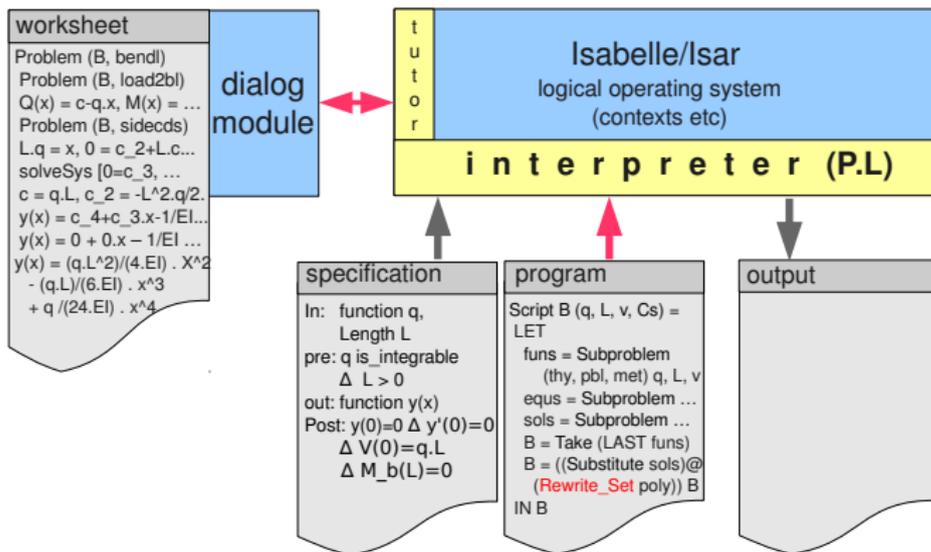
Tutoring

# Example: Tutoring step 5



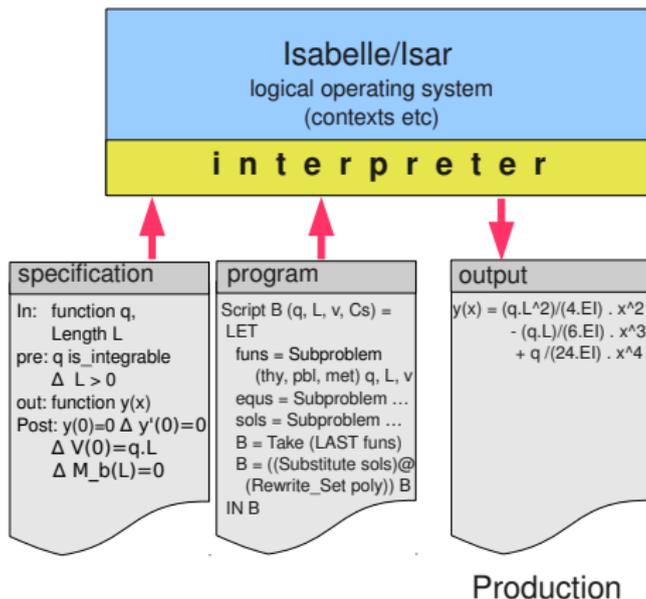
Tutoring

# Example: Tutoring finished



Tutoring

# Same problem: just get result



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# Dialog mechanically generated by CTP-technology

A calculation proceeded to a certain step (no. **3.**):

$$1. \quad \frac{d}{dx}(x^2 + \sin(3 \cdot x^4))$$

$$2. \quad 2 \cdot x^{2-1} + \frac{d}{dx} \sin(3 \cdot x^4)$$

$$3. \quad 2 \cdot x + \frac{d}{dx} \sin(3 \cdot x^4)$$

How can we guide the student to the next formula (no. **4.**)

...

$$4. \quad 2 \cdot x + \cos(3 \cdot x^4) \cdot \frac{d}{dx}(3 \cdot x^4)$$

...or some algebraically equivalent formula ?

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## Tutoring

Status quo  
Requirements

## Technologies

CTP  
Debug mode  
Transparent

## Summary and Conclusion

1.  $\frac{d}{dx}(x^2 + \sin(3 \cdot x^4))$
2.  $2 \cdot x^{2-1} + \frac{d}{dx} \sin(3 \cdot x^4)$
3.  $2 \cdot x + \frac{d}{dx} \sin(3 \cdot x^4)$

4. \_\_\_\_\_

# Dialog mechanically generated by CTP-technology

## Tutoring

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## Summary and Conclusion

$$3. \quad \begin{array}{c} \vdots \\ 2 \cdot x + \frac{d}{dx} \sin(3 \cdot x^4) \end{array}$$

$$4. \quad \underline{2}$$

# Dialog mechanically generated by CTP-technology

$$3. \quad \begin{array}{c} \vdots \\ 2 \cdot x + \frac{d}{dx} \sin(3 \cdot x^4) \end{array}$$

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# Dialog mechanically generated by CTP-technology

$$\vdots$$

3.  $2 \cdot x + \frac{d}{dx} \sin(3 \cdot x^4)$

4.  $2 \cdot x + \cos(3 \cdot x^4)???$

Input checked by a **prover**.

# Dialog mechanically generated by CTP-technology

$$\begin{array}{l} \vdots \\ 3. \quad 2 \cdot x + \frac{d}{dx} \sin(3 \cdot x^4) \end{array}$$

$$\frac{d}{dx} \sin(u) = \cos(u) \cdot \frac{d}{dx} \underline{\quad}$$

Provers identify and suggest theorems.

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### Debug mode

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# Dialog mechanically generated by CTP-technology

$$\vdots$$

3.  $2 \cdot x + \frac{d}{dx} \sin(3 \cdot x^4)$

$$\begin{aligned}\frac{d}{dx} \cos(x) &= -\sin(x) \\ \frac{d}{dx} \sin(u) &= \cos(u) \cdot \frac{d}{dx} u \\ \frac{d}{dx} x^n &= n \cdot x^{n-1}\end{aligned}$$

Provers operate on theories comprising theorems.

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Dialog mechanically generated  
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Provers use “matching” for fill-in gaps.

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Provers check, if a formula can be derived in a context.

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Summary and  
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# Demo Isabelle distribution

# Summary

General features and technologies like ...

- 1 **check user input** — comp. theorem proving (CTP)
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- 3 **explain steps** — human-readable knowledge

... provide for novel services in tutoring and authoring.

Austria can contribute significantly !

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## Conclusion ?

Possible Austrian contributions are ...

- ... **interpretation in debug-mode**, *ISAC*+Isabelle
  - TU Graz, Institute for Softwaretechnology
- ... **user-guidance** in single-stepping
  - TU Graz, IICM (H.Maurer)
- ... **Computer Algebra**
  - RISC Linz (B.Buchberger)
- ... **Dynamic Geometry**
  - Uni Linz (GeoGebra, M.Hohenwarter)
- ... **Didactics, teacher training, field tests**
  - ACDCA

Joint development of partners above ?

Start with a course by Ralph-Johan Back ?

## Conclusion ?

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